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A review of origami applications in mechanical engineering

Nicholas Turner, Bill Goodwine and Mihir Sen

Abstract

This is an overview of current research in origami applied to mechanical engineering. Fundamental concepts and definitions commonly used in origami are introduced, including a background on key mathematical origami findings. An outline of applications in mechanical engineering is presented. The foundation of an origami-based design procedure and software that is currently available to aid in design are also described. The goal of this review is to introduce the subject to mechanical engineers who may not be familiar with it, and encourage future origami-based design and applications.

Keywords

Mechanisms, design, complex systems, packaging, design research

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Introduction

The word *origami*, the ancient art of paper folding, combines the Japanese roots *ori*, meaning 'folded', and *kami*, meaning 'paper'.[1] Despite the art's rich aesthetic history, the vast majority of practical applications have come within the past 50 years. Advances in computer science, number theory, and computational geometry have paved the way for powerful new analysis and design techniques, which now extend far beyond the art itself. Even though mechanical engineering has always been concerned with devices that allow relative motion between components, which in a sense can be considered folding, the field of *mechanical engineering origami* is a recent development and it is leading to new and useful results. Folding linkages in one dimension, planar shapes in two dimensions, and polyhedra in three dimensions can now be efficiently designed and analyzed using origami concepts. This paper surveys the current state of research in origami applied to mechanical engineering. It briefly reviews *mathematical* and *computational* origami, disciplines on which most engineering rests, and overviews major applications that have been developed.

Several subdisciplines of origami that are useful in mechanical engineering have emerged over the years. *Orimimetrics* is the application of folding to solve engineering problems.[2] *Rigid origami* considers creases as hinges and models the material between creases as *rigid*, restricting it from bending or deforming during folding. *Action origami* is concerned with models that have been folded so that in their final, deployed state they can move with one or more degrees of freedom.[3] *Kinematic origami* is designed to exploit relative motion between components of an action origami model. *Kirigami* strays from traditional origami rules by allowing cutting in addition to creases, but provides a manufacturing advantage that is sometimes more suited to engineering applications. In many instances of so-called 'origami-based devices', 'kirigami' is the more appropriate label. It has found direct application in folding/morphing structures, micro-electromechanical systems, and cellular core structures for energy dissipation.[4–7]

First some terms that are common in origami must be introduced. A *crease* is a fold, either convex (mountain) or concave (valley). Collectively, all the creases make up the *crease pattern*. A *vertex* is a point where two or more creases intersect. The *degree* of the vertex is the number of creases emanating from that vertex. The *folded state* is the end result of some *folding motion*. A *pleat* is a fold that creates successive mountain and valley creases that are relatively close to each other. A *crimp* is similar, but involves some reverse-folding in a mountain–valley pattern. Figure 1 illustrates examples of crimp and

Department of Aerospace and Mechanical Engineering, University of Notre Dame, USA

Corresponding author:
Bill Goodwine, Department of Aerospace and Mechanical Engineering, University of Notre Dame, 376 Fitzpatrick Hall, Notre Dame, IN 46556, USA.
Email: bill@controls.ame.nd.edu

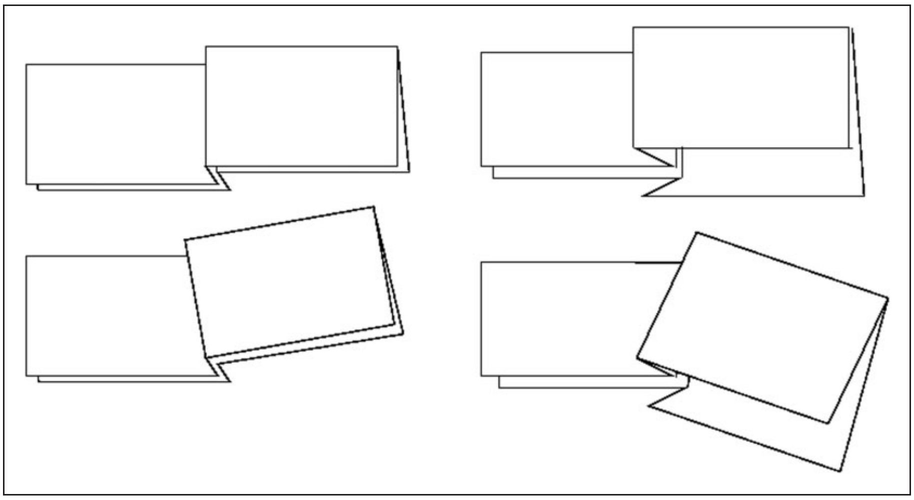

Figure 1. Left: pleat folds. Right: crimp folds.

pleat folds. These folds are used to create accordion and corrugated patterns used in a variety of applications.

The material used in an origami application is critical. Artistic origami uses paper which is an elastic material that prefers to be flat, but other materials are more useful for engineering. Creasing a sheet is essentially bending it beyond the yield point so it becomes plastically deformed. In surfaces with pleat folding, the physics will find an equilibrium among the forces that are at play in the crease patterns. This is important when three-dimensional (3D) structures are to be built from two-dimensional (2D) sheet material. If the material can be pieced together correctly and creased in such a way that each location on the material wants to locally bend and deform to the desired configuration, the 3D structures can easily be manufactured or self-folded.

The rest of this paper is organized as follows. Section 2 presents some of the mathematically based definitions and theoretical results in origami. While not all applications discussed subsequently rely heavily on these results, selected subjects from the state-of-the-art in origami theory the authors consider most relevant to mechanical engineering applications are reviewed because such theoretical results can provide a framework for more advanced developments in the field. Section 3 presents the most common crease patterns in origami relevant to mechanical engineering applications as well as their functions. In both the art as well as engineering, these crease patters provide a standard starting point and toolbox of functionality for origami engineers. Section 4 presents an overview of origami applications in mechanical engineering today. Section 5 discusses some of the practical issues in an origami design procedure, and Section 6 presents an overview of currently available software tools.

Selected mathematical background subjects on origami

Although much has been done and written about the the mathematics of origami, we will merely touch upon some of the salient geometrical, topological and optimization aspects relevant to this review. While it is the case that a very strong connection between the mathematic subjects in this section and the applications in Section 4 does not currently exist, one of the purposes of this review is to present the most relevant mathematical topics to help facilitate a development of a closer connection between the mathematical theory and mechanical engineering applications.

A *polyhedron* is any 3D surface composed of *polygons*, which are 2D flat surfaces with edges that are straight lines. Origami can be used to create *any* polyhedron from a flat piece of paper by folding.[8] Proving this involves folding a piece of paper down to a long, narrow rectangle. Next, the polygonal faces of the polyhedron that is to be modeled must be triangulated. This allows each resulting triangle on the face of the polyhedron to be covered. A zig-zag path, parallel to the shared edge with the next triangle and starting at the opposite corner, is used to visit each triangle on the polyhedron. *Turn gadgets*, which fold the strip onto itself with a mountain fold and folding the back layer over at the required angle, are used to create the path.

A path that minimizes overlap and covers each triangle only once is, in some sense, optimal for engineering applications. *Hamiltonian refinement* is a procedure that guarantees each triangle is only visited once through the use of a *spanning tree*, which is a graph that reaches all the vertices in the crease pattern.[1] This idealized path can be determined by drawing a line connecting the midpoints of each triangle. If this method does not result in the most efficient tree, then splitting each triangle into six smaller triangles will prevent revisiting any triangles as the entire polyhedron is covered.

The Huzita–Hatori (or Huzita–Justin) axioms are a set of rules in paper folding that define the full scope of single linear folds using points and lines. A *line* is either a crease in a piece of paper or the boundary of the paper. A *point* is an intersection of two lines. The axioms are complete in the sense that "these are all of the operations that define a single fold by alignment of combinations of points and finite line segments".[9] They are the foundation of logical constructions that can be used to form any regular polygon, and can also solve quadratic, cubic, and quartic equations, trisect angles, and determine cube roots.

The axioms (see http://origami.ousaan.com/library/conste.html) state that: (a) given two points p_1 and p_2, we can fold a line connecting them; (b) given two points p_1 and p_2, we can fold p_1 onto p_2; (c) given two lines ℓ_1 and ℓ_2, we can fold ℓ_1 onto ℓ_2; (d) given a point p_1 and a line ℓ_1, we can make a fold perpendicular to ℓ_1 passing through the point p_1; (e) given two points p_1 and p_2 and a line ℓ_1, we can make a fold that places p_1 onto ℓ_1 and passes through the point p_2; (f) given two points p_1 and p_2 and two lines ℓ_1 and ℓ_2, we can make a fold that places p_1 onto ℓ_1 and places p_2 onto ℓ_2, and (g) given a point p_1 and two lines ℓ_1 and ℓ_2, we can make a fold perpendicular to ℓ_2 that places p_1 onto ℓ_1. These operations describe simple folds and provide the basis of mathematical origami.

Flat-foldability is the property of a design that can be folded into a single plane with a thickness determined by the material. Making generalizations on global flat-foldability for multi-vertex folds is an NP-hard problem and remains open, however the single-vertex case is well understood. The crease pattern emanating from one vertex is defined by *n* angles between the creases, the sum of which is 360° for a flat piece of paper. Consider the crease pattern shown in Figure 2, which is flat-foldable.

For a single vertex to be flat-foldable, the following conditions must be satisfied.

- *Kawasaki's theorem* states that if the angles are sequentially numbered, then the sum of the odd angles must equal the sum of the even angles. This is evident in Figure 2, e.g. the sum of angles 1, 3 and 5 is equal to the sum of angles 2, 4 and 6 in the vertex on the left.
- *Maekawa's theorem* states that the number of mountains must differ from the number of valleys by ± 2. Every vertex in the crease pattern shown in Figure 2 satisfies this condition, where mountain and valley creases are black and gray respectively.
- The *degree n* must be even to satisfy Maekawa's theorem.
- For a complete origami design with multiple vertices, the crease pattern has to be *two-colorable*, meaning that each panel in the crease pattern can be colored with only one of two colors without having the same color meet at any border. This is again a necessary condition for flat-foldability of multi-vertex designs, along with each individual vertex satisfying the criteria above.

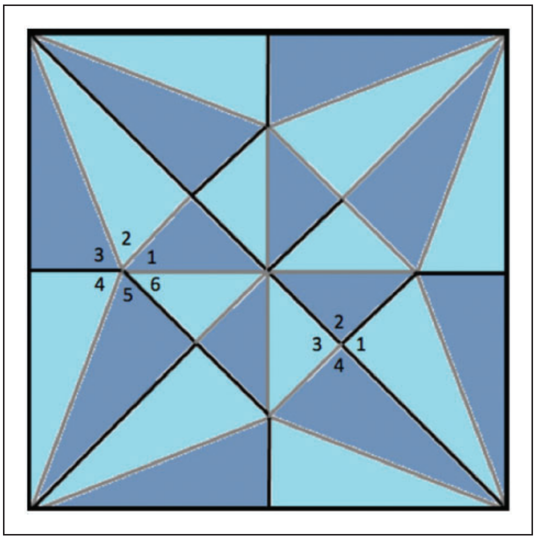

Figure 2. Flat-folding crease pattern. Mountain and valley creases are black and gray respectively.

Further generalizations of these theories have been made with the intent of imposing sufficiency and investigating global foldability.[10]

Design of folding patterns

To design an origami model, it is necessary to determine the crease pattern that will dictate the folds necessary to achieve the desired 3D form. An *origami base* is the first step in the folding process, and it is the foundation of every design. Several algorithms have been developed to design efficient crease patterns to fold bases, the most popular being the *tree method*.[11,12] A *limb* is a flap on an origami base structure and limbs are independently folded as the second step to add intricacy to a model. The tree method finds the folding pattern of the smallest possible square into some desired *uniaxial base*, and the projection onto a plane is the *shadow tree*. The base is uniaxial because the algorithm can produce bases with hinged flaps that can be folded to lie in a single vertical plane, resulting in a single-line shadow tree. *TreeMaker* is a program based on this idea and it generates the crease pattern necessary to fold any specific uniaxial base from the smallest square of paper. Consider folding a model of a cat, for example, where the base would consist of the body with six limbs (one head, one tail, and four legs). The shadow tree and uniaxial base for such a lizard can be represented by the design shown in Figure 3.

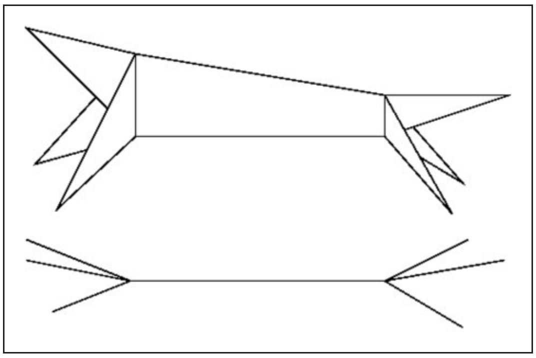

Figure 3. Top: uniaxial base. Bottom: corresponding shadow tree.

To generate the crease pattern from the smallest piece of paper, consider two points p_i and p_j on the paper before it is folded and the corresponding points s_i and s_j projected onto the shadow tree after the uniaxial base is folded. The distance between s_i and s_j, i.e. d_{s_i,s_j}, following the lines that make up the tree, or the *shadow path*, must be less than or equal to d_{p_i,p_j}. This leads to a key lemma in tree theory, which forms an invariant constraint that must be satisfied throughout every reduction made in the algorithm. The path between s_i and s_j is termed *active* if s_i and s_j are *leaves*, meaning they lie on the end of the limb such that only one point in the paper maps to the leaf on the shadow tree, and if $d_{p_i,p_j} = d_{s_i,s_j}$. Active paths are used to scale the shadow tree to its smallest size to fit on a piece of paper.

Let λ be a scale factor that satisfies $d_{p_i,p_j} \geqslant \lambda\ d_{s_i,s_j}$. The scale optimization step of the algorithm involves maximizing λ, or driving the *shadow paths* to become *active paths*. It is possible to design an efficient uniaxial base folding pattern by drawing circles centered on the leaf edges and with diameters equal to the length of the limb on the base in its folded state, where the circle defines the maximum possible reach distance after folding from the point at its center. This *disk packing method* uses non-linear optimization to arrive at a reasonable solution.

After optimizing these paths, the folding of the base can be determined. The points in the shadow tree, which occur at intersections and ends of lines, require incident creases in the crease pattern. Furthermore, the active paths in the shadow tree guarantee incident creases line up in parallel with a point in between, because this shortest path is the only way an active path can exist. In this way, the problem is broken up into subproblems, where uniaxial bases are made for each of the convex polygons determined by active paths or the boundary of the piece of paper. The constraints of bordering convex polygons must be satisfied and a universal construction tool is implemented to ensure realizability.

A *universal molecule* is used to fold a uniaxial base from any convex polygon of paper, where the vertices and edges of the polygon correspond to the leaves on, and the structure of, the shadow tree, respectively. These universal molecules are constructed as cross-sections of the polygon as it undergoes a constant shrinking process, where all the edges are kept parallel to the original edges. Higher dimensions in the uniaxial base, moving up from the shadow tree, correspond to increasingly reduced polygons. Simultaneously, the shadow tree is shrinking inwards to reflect the height of the leaves approaching the top of the uniaxial base. Tracking the trajectories of the original polygons' vertices provides the core creases of the crease pattern used to fold the desired uniaxial base, and they must all be mountain folds. Further detailed steps are necessary to complete the crease pattern design and guarantee that the base is achievable.

Rigidity theory

Understanding how linkages fold and unfold involves rigidity, a key concept in origami engineering. A *linkage* can be represented by a graph consisting of vertices and edges. A *configuration* is a linkage that includes coordinates for the vertices that satisfy each edge length. When a linkage folds or moves, it reaches many configurations and the complete set defines the *configuration space*. A linkage configuration is 'flexible' if it can move from some initial configuration in a non-trivial way (i.e. a motion that is not just a translation or rotation); otherwise it is 'rigid'. A planar truss is an example of a rigid linkage configuration. Testing rigidity of a given linkage configuration is a co-NP hard problem. For this reason, several assumptions and simplifications are made. There are mathematical constructions available to classify linkages as *generically* and *minimally generically* rigid. Useful applications are based on these studies, such as algorithms for building rigid linkage structures with the smallest number of links.

Combining the ideas of rigidity and linkages allows *locked linkages* to be mathematically defined as having a disconnected configuration space.[1] Linkages can be configured as either a *chain* or a *tree*. A chain is essentially a set of edges with a vertex and at least one other edge connected to it at each endpoint. A tree is a set of edges that can have branches of edges that end without reconnecting back into the inner set. 2D chains can never be locked, while trees can. On the other hand, all 3D chains and trees can be locked.[1] An unlocked configuration can be folded to any other configuration.

The study of *slender adornments* on folding structures is another important concept used to transition from theory to engineering applications. Linkage chains and trees consisting of just edges have been analyzed for unfolding and locking, but additional thicknesses or polygons have not been included. Slender adornments are arbitrary thicknesses or polygons that are attached to links in a chain or tree-like configuration. Consider taking a standard linkage chain and adding polygons on the chain instead of just the edges. These polygons will still be hinged at the vertices of the linkage configuration, but now instead of being concerned about overlapping edges, the non-crossing constraint becomes more difficult because the polygons will have less room to move before they intersect. Triangulating the polygons to model them as linkage configurations allows the same rules of rigidity and ideas of locked linkages to be applied.

Unfolding, folding and creases

To unfold a cube (or any polyhedron) consider cutting along the edges and then flattening the geometry into a plane. The reverse is the folding process. These ideas have many applications, for example folding any 3D shape out of a sheet of material requires knowing what shape to cut out of the original plane, and the necessary creases, so that it can be folded into the desired form without inefficient overlaps. Edge unfolding is a process where the cuts are only made along the edges of a surface. This is desirable if no visible seams can exist in the folded object. In general, cuts can be made anywhere on the 3D surface. In both cases, one piece of the sheet material with no overlapping regions is required. A non-convex polyhedron cannot always be edge-unfolded because the 2D shell often overlaps itself upon unfolding. General unfolding is an open problem. Note that *convex* and *non-convex* refer to the folded 3D form, not the unfolded 2D sheet. Figure 4 illustrates both cases of folding. The upper cube is deconstructed using edge unfolding and the bottom cube using a general unfolding process where the cuts are not constrained to the edges.

There are several methods used to achieve unfolding of polyhedra. *Edge unfolding* for both convex and non-convex polyhedra, *vertex unfolding*, *orthogonal polyhedra unfolding*, and *grid unfolding* are all methods that have been explored.[1] These ideas can be applied in engineering contexts when the problem of adding a finite thickness to the surface is addressed. The problem involves folding 2D surfaces into a 3D polyhedron. Exact coverage is desired, i.e. multiple layers are not allowed.

Instead of cutting edges (as in unfolding) the idea of *gluing* may be used to affix edges to one another during folding. Given a gluing pattern of a 2D shape, *shortest paths* can be determined before folding the 3D polyhedron. The shortest path on some surface S between two points i and j is the shortest of all curves connecting points i and j. Shortest paths always exist, but are not necessarily unique, and must exist as a straight line if the surface is unfolded to a plane.[1] A 2D polygon plus the gluings that bring it together create *metrics*, which are the distances points within a folded polyhedron are from one another. The *gluing* of a polygon involves matching

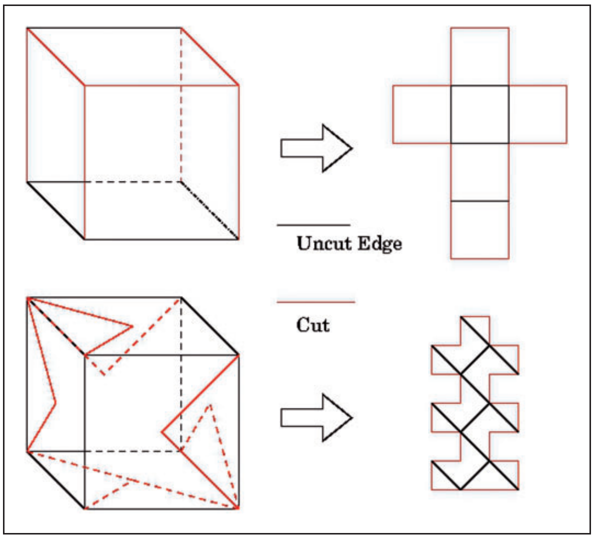

Figure 4. Top: edge unfolding. Bottom: general unfolding.

equal-length subsections of the boundaries of a 2D polygonal shape with one another in such a way that, when connected or glued, they form a polyhedron in three dimensions. The metric is determined by the shortest paths between any two points and will be convex if all points have zero or a positive curvature. Additionally, the metric should be topologically a sphere, that is to say that the gluing should be complete (no edges left out) and free of overlaps, which ensures no self-intersection of panels during folding. This idea is shown in the simple folding of a cube in Figure 5. Note the edges are paired together by their colors, and the gluing pattern, indicated by the lines connecting edges, does not overlap. The metric should also be polyhedral, meaning only a finite number of points have non-zero curvature. These properties, when satisfied together, can be termed *Alexandrov gluings*.

There are computer programs available that tell us how to fold any polygon with this type of gluing (see http://www3.math.tu-berlin.de/geometrie/ps/software.shtml). However, there also exist polygons that are *ungluable*. Additional algorithms exist for determining the number of possible gluings and methods used to achieve smooth foldings and unfoldings. These fairly abstract ideas have been applied to packaging and coverage problems in engineering. For example, determining the most efficient way to cover a spherical ball of chocolate with an initially flat piece of tin foil, which can be modeled by a polygon covering a polyhedron with minimal overlap.[13]

The idea of *fold and one cut* is an origami design tool that originated as a magic trick. Consider cutting out a simple five-point star shape. Starting from a square piece of paper, this shape would take 10 cuts to produce. Now imagine folding that square piece of

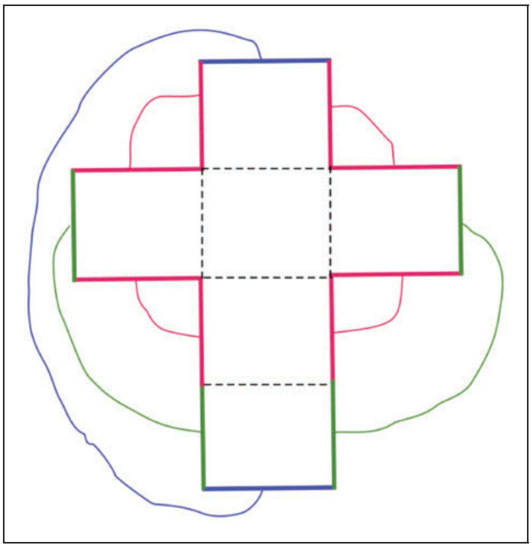

Figure 5. Polygon to fold a cube with gluing and metrics shown.

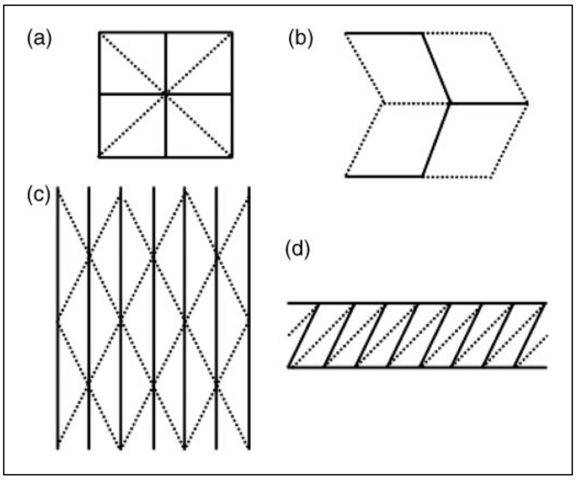

Figure 6. Common origami crease patterns, where the dashed and solid lines indicate mountain and valley folds, respectively (a) waterbomb base, (b) Miura-ori pattern, (c) Yoshimura pattern, and (d) diagonal pattern.

paper in such a way that one straight cut through the folded paper would produce the star upon unfolding. It has been proven that any *planar graph*, or shape made up of only straight lines, can be produced by this method, although some would take an unrealistic number of creases to achieve.[14] The basic method of the fold-and-one-cut origami design is to line up all of the edges of the desired figure onto one line that can be cut. The Huzita–Hatori axioms provide the foundation for these algorithms. This is a universal possibility as any set of line segments on a piece of paper can be aligned by flat-folding. Applications of these algorithms are rooted in manufacturing.

Common crease patterns

The *Miura-ori pattern*, *waterbomb base*, *Yoshimura pattern*, and *diagonal pattern* are all common rigid-foldable crease patterns.[15] Figure 6 shows these four crease patterns, which can be tessellated to form structures on any scale. The major features that distinguish these designs are that the waterbomb base and Miura-ori patterns can expand and contract in all directions, the Yoshimura pattern is capable of translational motion and the diagonal pattern allows for rotary motion.

Miura-ori pattern

The Miura-ori pattern is *auxetic*, meaning it exhibits a negative Poisson's ratio (i.e., when the pattern is stretched in one direction, the folded sheet expands in the orthogonal, planar direction), flat and rigid-foldability, and single-degree-of-freedom actuation. It was invented for use in space solar panels.[16] Figure 7 (left) illustrates the folding motion of a Miura-ori pattern. A variation of this pattern uses trapezoids rather than parallelograms and is used to create concave or convex structures, which are useful in architectural applications. The Miura-ori pattern has been extensively used in engineering.

Waterbomb base

The waterbomb base has applications in smart materials and actuation due to its simple geometry and multiple phases of motion,[17] and is commonly used as a base for more complicated designs. The folded states are shown in the center and right in Figure 7. It is easily manufactured, has a transferable crease pattern, is readily scalable, is rigid-foldable, can be expanded for different designs, and can be actuated in three difference phases of motion.[17] The waterbomb base is also flat-foldable and when tessellated it creates an axial contraction segment with a negative Poisson's ratio between the radial and axial directions.

Yoshimura pattern and diagonal pattern

The Yoshimura pattern is a tessellation of diamonds, with either all mountain or all valley folds along diagonals. The curve of the sheet after folding, which yields the radius of a cylinder or curve, depends on the shape of the diamonds in the pattern. A hexagonal variation of this crease pattern is also possible if additional folds are made along the diagonals of the diamonds. The diagonal pattern is also common in folding cylinders. However, instead of contracting in a translational manner, it rotates as it collapses.[15] It was first observed as the natural reaction when torsion was applied to a cylinder.[18] The crease pattern is made up of parallelograms that are folded in one direction along their diagonals and in the opposite direction along their parallels. The valley folds of a Yoshimura pattern form a planar polygonal line,

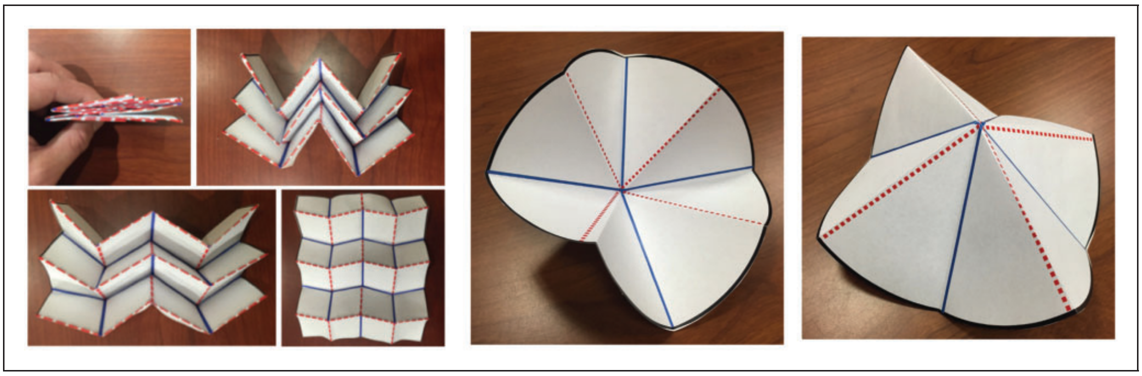

Figure 7. Left: Miura-ori pattern. Center and right: waterbomb base in two stable equilibrium positions.

while the valley folds of a diamond pattern form a helical polygonal line.[19]

Mechanical engineering applications of origami

This section outlines the major applications of origami in mechanical engineering. While there is not a complete disconnect between the theory and applications, the dependency of applications on the theory is not as extensive as it is in more mature mechanical engineering disciplines. Where appropriate we point out the connections and at the very end of this section we provide an overview of the relationship between the theory and applications.

Applying origami to engineering

Paper, which is assumed to be two dimensions in most mathematical studies, is not the material that is used in the vast majority of engineering applications. However, it is important to study and understand how paper folds between creases in origami in order to extrapolate these results to materials that are used in engineering. Earlier, it was assumed that the faces of the paper stayed straight during folding. However, this is not necessarily true because paper is flexible.

To explain how the surface folds, define *Gaussian curvature* as the product of the minimum and the maximum curvature at any one point on a 3D surface. It is negative for saddles, positive for convex cones, and zero for intrinsically flat surfaces. The total Gaussian curvature never changes during folding. Folding a piece of paper will always result in a form with zero curvature and the minimum curvature will locally be zero at every point. This explains how slices of pizza are most effectively handled by depressing the middle of the crust to give some curvature to the slice and supporting the length of the pizza, which is now restricted from folding.

One major challenge in the transition from theoretical origami to engineering is the addition of some finite thickness in the materials. In the majority of mathematical results that have been developed, 2D surfaces, with zero thickness, are assumed. Several methods for adding thickness have been proposed and they all involve some adjustment at the hinges, or creases. Essentially, the edges in any folding design can be hinged together at valley creases. The main problem is when there are several fold lines at one vertex. There can no longer be concurrent edges and the edges become over-constrained. There are ways to use symmetry at each vertex and achieve a workable design. There are also slidable hinges that allow edges to slide along the faces of connecting panels.

One way to solve the over-constrained issue, instead of moving the hinges to valley folds, is to trim the volume of the edges on the valley sides. This allows the vertex to flex in a way that the edge does not intersect itself. Figure 8 shows examples of these methods. Another hurdle in many origami engineering applications is the cost and the time spent folding, which presents a barrier to applications where folding may be introduced. Additionally, durability must be achieved as engineering applications will likely require repeated folding and unfolding. The following are some principal areas of mechanical engineering applications of origami.

Delivery, packaging and storage

Folding can be used to improve the performance of devices that operate in a limited space. For this reason, devices in this area generally exist in either a folded or unfolded state, and do not display final motion in either orientation.

Packaging. Most engineering applications make use of materials that are less flexible than paper and are approximated as rigid. Mathematical solutions can be implemented in many different engineering applications. Packaging for consumer goods provides a widespread example of rigid origami, including automated packaging folding processes and designing the most efficient packaging.

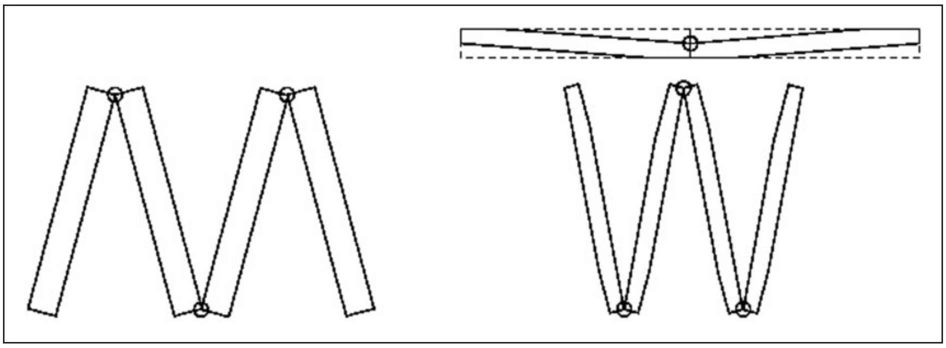

Figure 8. Volume trimming in thick origami.[1]

One recent example of origami in manufacturing packaging is flat-folding rigid shopping bags.[20] The solution allows the bottom of the carton to remain rigid and can be applied to shoppings bags with various dimension ratios and thicknesses. The new crease pattern is a variation of a traditional pattern used for folding bags, but the upper and lower portions are separated with a horizontal crease around the bag. It is relatively easy to show that the lower portion, including the base, is rigid-foldable. To achieve a working design, the bag is split into four identical sections, centered at each corner, and only one section is analyzed due to symmetry. The vector-based approach ensures the bag is rigid-foldable by proving each panel in the structure remains planar and connected to neighboring panels throughout the entire folding motion. The only design variable is the choice of an angle that dictates a crease on the side of the bag from the horizontal. This variable has maximum and minimum allowable values, determined by the ratio of the height relative to the depth of the entire bag. The only other restriction in the design is providing sufficient width to the bag to ensure that the top corners of the bag do not intersect during the folding motion. Due to the highly non-linear nature of the conditions leading to rigid-foldability, solutions are found numerically.

Another analysis of carton packages focuses on the tuck-in operation that is commonly used to secure a lid.[21] An equivalent mechanism to a carton appears here, where the creases are joints and the panels are links. The stiffness of the links is important because the tuck-in operation is not possible with a single rigid link. To achieve a design with rigid links, the carton lid is decomposed into an equivalent three-link mechanism that allows the flap to be tucked in. The three smaller links can be considered rigid and the corresponding kinematic equations are derived that allow the flap to be guided into the slot. The angular position of the end of the flap is determined and the necessary torque to drive this mechanism can be found. A robotic manipulation device is then proposed to carry out the tuck-in of the carton flap. Motors are chosen for this task to allow the tuck-in operation to be completed within 1–2 s, allowing the machine to compete in the packaging market. An inverse kinematics approach is used to analyze the trajectories in this complicated packaging problem.

Shipping containers. The transportation of empty containers is inevitable in the shipping industry, and several attempts have been made to manufacture foldable shipping containers.[22,23] This can be formulated as an origami engineering problem. Simplicity and durability in unfolding and folding of the containers is a must and lightweight materials should be used to keep the tare weight down. Although actuation for folding is commonly used in engineering origami, manual unfolding and folding may be more appropriate in this case to reduce costs and to retain the robustness of the design. A similar crease pattern as used in the rigid and flat-folding shopping bag may be helpful in this application, given the robustness of that solution to various dimension ratios.[20] So far two major foldable containers have been introduced into the market,[22] but neither were commercially successful as they had higher tare weights and were significantly more expensive than the standard containers. The search for a foldable container continues.

Optics. Origami has been used to determine the most effective way of 'folding' long-focal-length optics into small spaces, a field of study called *optigami*.[24] The general idea is to reflect light, using mirrors, many times to create high-resolution, large-aperture cameras with reduced thicknesses.[25] This approach reduces the size and weight of imaging devices currently in use, which finds application in surveillance, telescopes, and cell phones for example. Studies are available on nanoscale origami used for 3D optics,[26] and on photo-origami-bending and folding polymers to program optical fields into materials.[27] Another application of optigami is the *Foldscope*, which is a flat microscope that can be used to achieve 2000× magnification and submicron resolution.[28] This device can be assembled in about 10 min from a flat sheet of paper and several other small components and results in an optical microscope costing less than US$1.

Space. Rigid origami has for a long time been applied in space to the deployment of solid solar panels[29] and inflatable booms for deployable space structures.[30] A benefit of rigid origami is its scalability and single-degree-of-freedom actuation. Origami fold patters have inspired mechanical linkages that exploit the motion of a single vertex and extend this kinematic behavior to a patterned system of vertices, resulting in a mechanism that exhibits single-degree-of-freedom motion. The Miura-ori pattern (Section 3.1) was first introduced for the deployment of solid solar panels in space and continues to be used.[31] Figure 7 shows the folding pattern in three folding positions. This pattern is ideal for folding solar panels because it satisfies the constraints of rigid and flat foldability.

Biomedical devices. Biomedical applications represent a growing area of interest in origami engineering devices designed for delivery in constrained spaces. To date, 3D biomedical structures such as encapsulants, particles, scaffolds, bioartifical organs, drug delivery devices[32] and minimally invasive surgical tools have been explored.[33] New self-assembly techniques, actuated by heating or a chemical stimulus, are being used to complement existing 3D tissue fabrication and patterning methods. Self-folding can occur in hingeless 2D planar structures, resulting in curved structures, and also in hinged micro- and nanoscale structures that result in 3D polyhedra. When temperature is used to actuate folding, polymers can be used in the planar materials, and the edges of the polymer will fuse together after self-folding, creating a mechanically robust device.

Surgical devices can benefit from self-folding and tetherless tools that allow greater access to hard-to-reach areas within the human body and truly non-invasive surgery. Origami-inspired forceps have been developed based on spherical kinematic configurations of origami models, and the use of shape-memory alloys (SMAs) will promote research in this application. A new design for a stent graft has been inspired by origami and can be deployed in a blocked or weakened artery or intestine, where it is unfolded to unblock the area in a minimally invasive way.[34] The SMA is deployed when exposed to body temperature. It also includes an integrated enclosure that prevents restenosis, which is the blocking of a stent by subsequent tissue ingrowth through openings in the meshes. A common triangular mesh folding pattern is used to create a cylindrical structure that folds down, both laterally and in diameter, and then can be deployed and re-opened once in position. Figure 9 shows the crease pattern of the origami stent, which is a tessellation of the waterbomb base (Section 3.2) and is connected at opposite edges to form a cylinder.

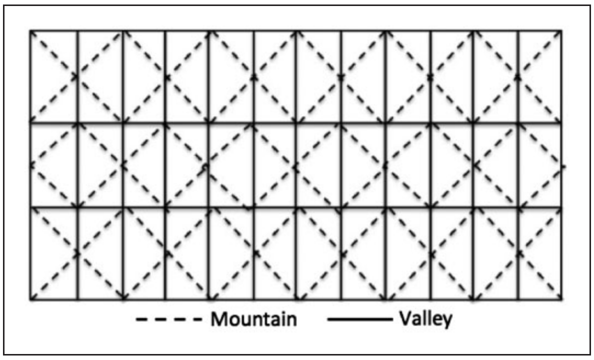

Figure 9. Origami stent crease pattern.

Other storage applications. Several other applications of origami in storage and delivery must be mentioned. Automobile airbag design involves folding an airbag into a compact state that allows it to be rapidly unfolded in milliseconds. The 3D shape of the airbag is critical in the effectiveness of the device, and concepts from rigid origami and flat foldability are used to design the creases that flatten the airbag. Classic origami geometries are being used to create antennas and other electronics designed to collapse down to small sizes. Several designs, following from the accordion or pleat fold bases or variations of the Yoshimura pattern (Section 3.3) for collapsible cylinders, have been developed.[35,36] The frequency of each antenna can be tuned based on its height, providing a device that can be stored in a pocket and then easily deployed for long-distance communication needs.

An origami-inspired kayak can be manually unfolded from a $32'' \times 13'' \times 28''$ box to a $12'$ long $\times$ $25''$ wide fully deployed vessel in just a few minutes.[37] Rigid origami concepts and the Miura-ori pattern (Section 3.1) have also been used to design lithium-ion batteries that fold, bend, and twist to provide deformable energy storage devices.[38] Applications of this technology include flexible displays,[39–41] stretchable circuits,[42] hemispherical electronic eyes,[43] epidermal electronics,[44] and conductive textiles.[45]

Manufacturing

Origami has been the inspiration for many applications within mechanisms used in manufacturing. Fundamental origami concepts have been used to study kinematics of mechanisms, simplified processing, automated folding, and optimized self-folding. This section outlines applications of origami in mechanical engineering related to these subjects. Generally, devices for this type of application exhibit some final motion in their folded and/or unfolded state and thus can be classified as *action origami*. This is a unique characteristic as compared to applications categorized in Section 4.2, which are generally designed to fold and unfold into static states.

Self-folding and self-assembly. It is common for engineering systems to require complex and time-consuming manufacturing processes and deployment methods. However, nature provides many examples of self-folding structures that can be quickly fabricated and assembled, which have inspired novel engineering methods. Self-folding ''automates the construction of arbitrarily complex geometries at arbitrarily large or small scales'',[46] and, by doing so, can provide innovative solutions allowing for faster manufacturing processes, reduced material usage, reduced part count, and improved strength-to-weight ratios.

A variety of self-folding mechanisms have been explored to date. A highly referenced systematic study of self-folding, without any direct mention of origami has been compiled.[47] Examples of self-folding origami engineering include mesoscale structures that fold when actuated by lasers and magnetic fields,[48,49] pop-up mechanisms that use micro-electromechanical systems techniques,[50,51] SMAs that actuate self-folding sheets of programmable matter,[52] single-use shape memory polymers (SMPs) that self-fold into target structures using selective light absorption with patterned inks,[53,54] and self-folding robots and structures that rely on SMPs with resistive heaters.[46,55–58] A methodology has been developed to coax thin membranes into collapsing into 3D forms on microscopic and smaller scales.[59] Using a triangular network of creases, a thin membrane can achieve a variety of desirable forms that include flat sheets, partially crumpled or collapsed into a compact state. A Brownian motion simulation is used to analyze the dynamic collapse of a membrane, which employs the same crease pattern as the origami stent.[34]

Robotics for origami. Manufacturing origami-inspired products requires robots capable of bending and folding materials. Mathematical models and origami concepts are largely applied to linkages and mechanisms, which are directly used in robotics. Though this is an essential part of the application of origami to mechanical engineering, we will not devote much space to it here because robotics and its applications have been extensively considered elsewhere in the literature. Manipulating paper to fold traditional origami exemplifies many of the current challenges faced in dextrous manipulation and flexible object manipulation in the field of robotics today. For this reason, robots that fold traditional paper origami have been used to uncover and explore the difficulties associated with the manipulation, modeling, and design of foldable structures.[60]

Origami has also inspired the design of a new class of robotic systems specifically designed for new rapid and scalable manufacturing processes. Building sophisticated 3D mechanisms from a 2D base structure incorporates elaborate folding patterns that can execute complex functions through the use of actuated hinges or spring elements. An origami approach that will significantly drive further advancement in printable robotics has been identified.[15] Hardware limitations are currently constraining the mobility, manipulation capabilities, and manufacturing of robots. Complications also arise in software as an algorithm capable of manipulating the paper in the correct sequence with the least number of steps is desirable. By employing an origami approach, 3D mechanisms capable of complex tasks can be printed on 2D planar sheets and then subsequently folded into some final state. This is a low-cost and extremely fast method for designing and fabricating new robots with expanded capabilities. An additional benefit is that these robots have the potential to be folded back down to a planar state for storage and transportation.[15]

Mechanisms. Studying mechanisms is an area of interest in mathematical origami. Again, we will have little to say about this because of the extensive literature that currently exists on the subject. Origami can be directly modeled as a compliant mechanism, where the creases act on pin joints and allow motion.[61,62] *Lamina emergent mechanisms* (LEMs) are a subset that have an initial flat state and motion emerging out of the fabrication plane, which is analogous to folding origami from a flat sheet of paper. The *pseudo-rigid-body model* (PRBM) is a model representing compliant mechanisms as rigid-link mechanisms with torsional springs at their revolute joints.[61] Graph theory offers a common ground between mechanisms and origami as the two can be abstracted to a common graph.[2] This allows mechanisms and origami to be understood and analyzed using similar conventions and mathematical techniques.

Spherical mechanisms are often used to study kinematic origami models. The motion of the origami model is traced down the folds to the center of each spherical mechanism. In this way, a vertex in origami is equivalent to the sphere center of a spherical mechanism.[60,63,64] Once the vertex is located, the folds that are in motion can be identified and these folds map to links in the corresponding spherical mechanism. In most origami models, artistic features disguise the underlying mechanisms. However, graph theory and simplified origami models have been made to classify the types of spherical mechanisms used today in action origami. The classification scheme is purposefully generalized—neglecting the number of links, link lengths, link shapes, and internal angles–to allow for flexibility so that the same fundamental mechanisms can be applied to provide motion for very different models.[3]

Pop-up mechanisms. Pop-up mechanisms offer an interesting area of study relating mechanisms to origami. They deviate from the traditional rules of origami and even those of kirigami, by allowing cuts *and* the use of glue to attach more than one piece of paper together in a design. However, they do exhibit the concepts of flat-foldability, making them very interesting and useful to study. Commonly seen in children's books, pop-up mechanisms involve a 3D structure self-erecting by the action of opening one crease. Several principles involved in designing pop-up mechanisms have been studied with the intent of closing the gap between art and engineering applications. Further understanding of the kinematic principles at play in these complex mechanisms provide insight into potential applications beyond paper engineering, such as airbag folding, sheet metal forming, protein folding, packaging, and other single-degree-of-freedom applications.[65]

Industrial origami. Origami-structured industrial products start from a flat sheet of material and then are folded into some final shape. This method offers a low manufacturing cost and provides advantages such as rigidity in the folded state and flat-foldability for storage and transportation. The folding process also introduces strength in the material. One application that has been explored is manufacturing sheet metal such that it can be folded to create the frame of consumer appliances[66]. This study analyzes the material properties and forming process required to create large-scale metal products in industrial applications.

Electrical devices. Print and self-fold electrical devices on the millimeter scale have been created using a polyester film coated on one side with isotropic aluminum (a metalized polyester film, MPF). When globally heated, the internal stress due to the contraction in the sheet is transformed into a folding torque.[67] The 2D film is folded into 3D electric devices; specifically a resistor, a capacitor, and an inductor. Particular resistances can be attained by varying the material's geometry, which is achieved through folding different lengths of the MPF. Electrically isolating two MPFs results in a capacitor, and the capacitance can be controlled through folding by altering the surface area and spacing between plates. An inductor in the form of a coil can be used as an actuation mechanism, where the inductance is determined by the number of coils and the coil geometry, which can be controlled through folding. Specifically designed folding patterns were defined by laser machining to manufacture the MPFs necessary to fold into these components.

Dielectric elastomers (DEs) exhibit favorable material properties for folding and unfolding and, for that reason, have been used to actuate origami structures.[68] DEs are low-modulus electroactive polymers that use an electric stimulation to cause a Maxwell stress that drives a lightweight polymer, generating mechanical motion. DEs have high specific elastic energy density, large strain response, fast response time, high actuation stress, and high electromechanical coupling efficiency.[69,70]

Structures

The following are some applications of origami that appear in mechanical engineering structures.

Deployable structures. Several origami concepts, such as flat-foldability and single-degree-of-freedom actuation, find applications in the design of deployable structures. Some of these applications use a cylindrical shell that collapses into a 2D plane under torsional loading, which naturally creates the diagonal pattern (Section 3.3) shown in Figure 6 and discussed previously in this review. Figure 10 is a series of images displaying the folding motion of the diagonal pattern. Experiments show that the analysis is inaccurate for shorter cylinders where the boundary conditions, either assumed to be simply supported or clamped, interfere with the buckling pattern.[18] In addition, membrane compression is maximized at some oblique angle to the axial direction. Optimization of the truss design for folding and unfolding with minimal energy input for more versatile deployable structure applications remains to be done.

Figure 10. Foldable cylinder based on twist buckling.

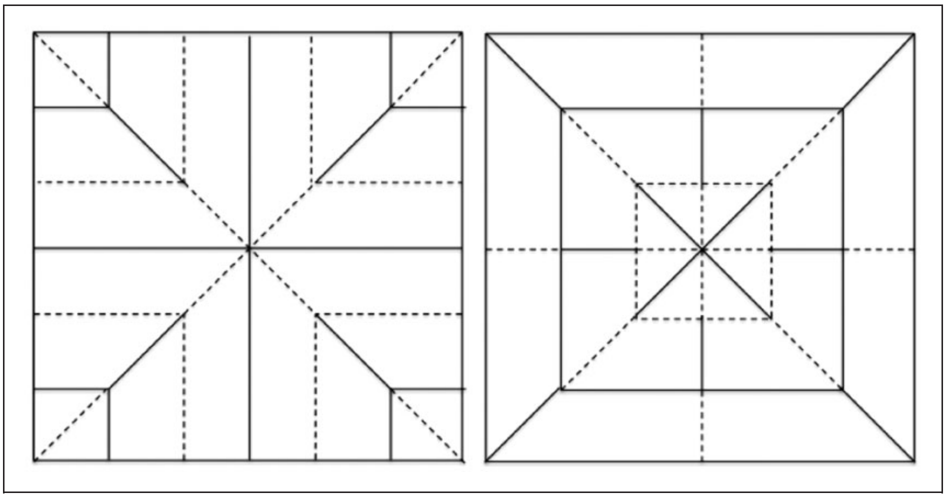

Figure 11. Leaf patterns. Left: leaf-out. Right: leaf-in.

Biomimicking has also inspired the design of deployable structures.[71,72]

Tree leaves were the inspiration behind *deployable membranes*. Leaves have biologically evolved with a balance of flexibility and rigidity, which allows them to fold in the wind to decrease drag and damage, while simultaneously being strong enough to support their own weight along with occasional other loads. Veins and midribs in leaves act as stiffening members, or links, supporting flexible membrane panels. The geometric study of tree leaves, including relating the folding pattern to the Miura-ori pattern (Section 3.1), has been carried out.[73] As defined in the study, a leaf-out pattern is one in which the 'leaves' are directed away from the center of the polygon and vice versa for the leaf-in, as shown in Figure 11. Variations and combinations of several known leaf patterns are explored in order to produce deployable structures, which include solar panels, antennas, solar sails, folding tents, and roof structures.[74]

Deployable shelters, used primarily for disaster relief and military operational bases, represent another origami-inspired application.[75–77] The key design features of these structures include lightweight frames, high volume expansion ratios, and rigid-foldability. Accordion or pleat crease patterns, and variations thereof, have been used in previous studies. More intricate crease patterns, with more optimal folding behaviors, can be applied in the future.

Two techniques are used to analyze the kinematics of origami. The *unstable truss model* uses the configuration of vertices of the structure to constrain the motion of the structure by preserving the length of the links in between vertices and the diagonals of the facets. This ensures that both the links and the panels in the model are rigid. The second method is the *rotational hinges model*, which uses the rotational angles of edges to represent the structure. To constrain the motion, this model forces closed loops to remain intact during the folding motion.[78] A mathematical approach to the rotational hinges model using rotational matrices has been proposed.[79] The structural configuration is represented by the folding angles contained along some closed strip of facets in the model that remain connected during the folding motion.

Architecture. There are several advantages to a rigid-foldable origami design in architectural applications. These include: (a) a watertight, continuous surface is ideal for constructing an envelope of any space, roof, or facade; (b) a rigid origami model offers a purely geometric mechanism that can be realized at any scale because it does not rely on the elasticity of the materials and is not significantly hindered by gravity; and (c) the transformation of rigid origami from an unfolded state to a final configuration is controlled by a smaller number of degrees of freedom, which enables semi-automatic deployment.[78]

Applying rigid origami to designs in architecture, the geometry in kinetic motion is analyzed to discover generalized methods through which additional rigid origami designs can be created and existing designs can be modified. Studies using the Miura-ori pattern (Section 3.1) have been made.[4,80,81] Using common rigid origami patterns, two basic approaches to achieve rigid-foldability have been proposed. One approach is based on triangulated patterns where the degree of freedom is determined by the number of elements on the boundary. Another involves quadrilateral patterns that provide single-degree-of-freedom motion. The latter has been employed to demonstrate an architectural application of rigid origami for the design of a foldable hallway connecting two offset and uniquely sized openings between buildings.[78] The design begins from a known regular quadrilateral origami pattern, which is modified to satisfy several constraints through optimization using the Newton–Raphson method. In the end, a variational design is found and a method for thickening the panels for manufacturing is proposed.

X-Ray machine shroud. Origami-adapted structures have been devised to cover the non-sterile extension C-arm of an X-ray machine in an operating room. Plastic drapes have traditionally been used, but they were not durable and had to be replaced every time the movable device entered and exited the sterile field. To achieve a sturdy design that met the sterilization needs but did not limit the movement and position of the arm, an origami-based design was implemented.[82] The design uses a slightly modified version of the Miura-ori pattern (Section 3.1) to account for the contours of the arm. Essentially, the shroud design covers the entire arm, creating a barrier, while it rotates in and out of the sterile field. This approach saves time and money associated with repositioning the machine.

Energy absorption. Compliant mechanisms modeled by origami have inspired several designs for energy absorption and impact force distribution. The Miura-ori folding pattern again finds utility in energy dissipation through crushing or plastically deforming its shape. This is due to its single-degree-of-freedom motion paired with a negative Poisson's ratio.[83] This unique mechanical property is helpful in absorbing energy in the deformation of the folds and distributing an impact force throughout a structure.[84] Compliant mechanisms are used to analyze paper origami and origami-adapted engineering designs, and the best have been shown to have a high yield-stress-to-elastic-modulus ratio.

Another origami pattern used in energy absorption, as well as deployable and foldable structures, is the Tachi–Miura polyhedron (TMP) bellows, which is a rigid-foldable, approximately cylindrical structure composed of two modified Miura-ori rectangular sheets attached at the two longer edges. In an analytical model,[85] the flat facets of the TMP remain flat during the folding motion and all deformation occurs strictly along the crease lines, so the mechanical work done by the external force can be equated to the bending energy along the crease lines, with some energy dissipation.

Sandwich core structures. Sandwich core patterns are used in many structures, including aircraft and wind turbines, to increase strength-to-weight ratios. Conventional methods include hexagonal honeycombs, but these designs possess positive Poisson's ratios, which result in the structure bending into a saddle-shaped curve when stressed in one plane. *Foldcores* are origami structural sandwich cores created by folding a planar base into a stronger 3D structure. A design has been suggested that exploits the advantageous properties of honeycomb cores while avoiding the disadvantages of humidity accumulation.[86] The foldcore is fabricated by carving or stamping the creases onto a sheet material and folding along these edges. A zig-zag pattern has been used to create a core and aramid paper has been tested and simulated as a base material to determine its mechanical properties and performance while folding.[86]

Kirigami has inspired a new graded conventional or auxetic honeycomb core with higher density-averaged properties, including compressive modulus and strength.[6] To produce complex geometries capable of achieving an auxetic honeycomb core, kirigami has been used to create a cellular tessellation with improved performance over traditional honeycomb cores. Similarly, a lattice auxetic pyramidal core has been developed.[87]

Graphene folding. Graphene is an engineering material composed of a single layer of carbon atoms bonded in a repeating hexagonal pattern. The material is so thin that it can be approximated as 2D and can easily fold when subjected to external stimuli. It is also extremely strong and conducts heat and electricity with great efficiency. Studies have explored the folding behavior of mono- and multi-layer graphene sheets.[88] The introduction of other shapes into the hexagonal network, including pentagons and hexagons, can influence the way a graphene sheet folds and the 3D forms it can achieve, which directly influence its material properties.

This research provides a starting point for *graphene origami* which can be used to engineer carbon nanotubes, cones, graphene wraps, and other structures that exploit the many favorable characteristics of graphene at small scales. Programmable graphene origami is of interest and is used to create nanoscale building blocks. A self-folded tri-layer graphene specimen was analyzed using a non-linear continuum mechanics model based on beam theory along with molecular dynamics simulations.

Curved-crease origami. Traditional origami, and virtually all engineering origami, is concerned with straight line creases. Several aesthetic and purely mathematical explorations of curved-crease structures exist and the geometric mechanics of these structures have been explored.[89] To form a foundation for advanced analyses of curved crease structures, which may provide advantageous structural properties in some applications, a simple example of curved-crease origami consisting of a circular strip with a single crease along its center has been folded to form a 3D buckled structure. The angle of the fold, radius of the circle and properties of the paper are used to quantify the shape and analyze the folded structure.

It has been shown that a cut annulus with a concentric, circular crease remains flat after folding, while the same crease in an uncut, complete circular annulus forces the form to fold into a saddle due to the elasticity of the sheet and the in-plane stresses created by the crease. In either case, the folded state is driven by minimizing the total elastic energy from the sheet and the fold, which are derived and expressed in terms of the curvature of the paper and the torsion within the crease. The paper is treated as a developable surface, which restricts stretching between the creases and in this way models rigid origami. A triangular mesh model for the curved structure, where each edge was treated as a linear spring, was used to minimize the energy of the system using a direct numerical approach. The study continues to define geometric constraints associated with the maximum dihedral angle of the fold relative to the curvature and torsion. A more advanced study involving a series of concentric curved folds (an example of *pleat folding* due to the altering mountain–valley pattern) has furthered the structural understanding of curved-crease origami.[90] In this case, the resulting shapes include a saddle shape, similar to the case of a single crease, as well as a helical form.

Additional applications

The above three application spaces encompass the majority of current research in origami related to mechanical engineering. Another origami application that does not fit easily in the previous sections is the use of mathematical origami in computer graphics to enhance the rate at which data is sent through a computer in animation.[91] In *tunable metamaterials* origami is used to adjust the spacing between a series of split-ring resonators placed on a folded surface, resulting in a range of resonance frequencies.[92]

Applications summary

As is clear from the length of this section, the applications of origami in mechanical engineering are diverse, interesting and important. Figure 12 illustrates the relationship among the origami properties such as flat-foldability, the folding patterns and the applications which make use of them.

Origami-based design procedures

Now that a broad overview of origami applications in engineering has been presented, the advantages and usefulness of origami-based design becomes more clear. There are four basic properties that must be considered in converting a crease pattern into a functional engineering design.[93]

1. Rigid-foldability is a property of a crease pattern. If the crease pattern is proved to be rigid-foldable, then crease characterization (step 2) can occur. If not, several secondary creases must be added or boundary material must be removed to allow for rigid-foldability. Non-rigid designs are analogous to over-constrained structures. By adding additional joints, the number of degrees of freedom in the design can be increased to allow the crease pattern to be rigid-foldable.
2. The surface need to be classified as *uninterrupted continuous* and the creases must be characterized

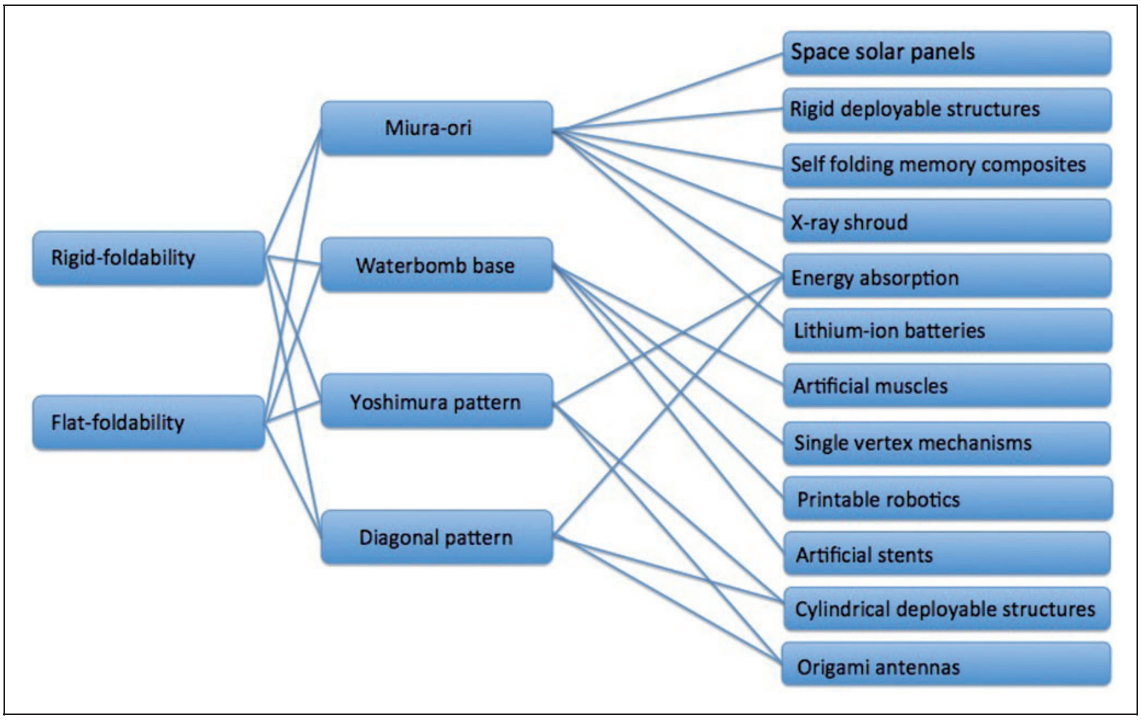

Figure 12. Relationship among fold properties, fold types and applications.

by the degree to which strain energy storage is desired. An uninterrupted continuous surface is a closed surface without holes. If this is a desirable characteristic in the application, then scoring, etching, heating, or mechanical folds can be used to create the creases. If no constraint for an uninterrupted continuous surface exists, then perforations can also be used.

3. Strain energy, stored elastically in the creases, is the next factor. Depending on the design constraints, differing amounts of strain energy storage are desirable. Heavier perforations or deeper scoring are methods used to modify the cross-sectional area at the creases and raise the crease hinge index, improving the hinge behavior. If more strain energy storage is desired, then a lower hinge index material, which includes polymers and metals, can be used. The material properties and dimensions dictate the strain energy capacity and relate to the crease characterization. The material choices affect the rigidity of the panels and the deflection at the creases. The appropriate stiffness or compliance, depending on the application, can be designed by setting the correct thickness of each of these parts. Accommodating the thickness is another consideration. Material selection is not limited to monolithic materials because composites and sandwiched membranes have been used.[29]
4. Once the material is chosen, the manufacturing method is the last step in the design. Creating creases in the material is a challenge, and computer numeric controlled (CNC) methods likely offer the most flexibility at this time. Several CNC methods have been discussed,[93] including plasma cutting, abrasive water jet cutting, laser cutting, incremental sheet forming, and nibbling. Folding the final product can be achieved using various methods, and automated folding, using a robotics approach, has been considered.[60]

Another origami design procedure has been proposed but focuses on *kinetogami*,[94] which allows cuts, as in kirigami, but also relies on folded hinges that exist across *basic structural units* (BSUs). BSUs are structural polyhedral links with empty volumes that are modeled as rigid bodies and used as building blocks to create 3D forms. The design procedure, which allows for manufacturing 2D sheets that can continuously fold into 3D forms, involves:

1. designing a set of basic BSUs formed from tetrahedral, cubic, prismatic, and pyramidal components;
2. synthesizing each BSU's crease and cut pattern to create a single 2D pattern;
3. altering the design parameters to provide reconfigurability;
4. extending one BSU unfolded pattern along a linear path on a sheet, and folding each pattern into a string, which is adopted based on previous research, proving that linear chains of polygonal models can be folded into arbitrary 3D shapes;
5. threading the string through the correct Eulerian cycle to allow for folding and reconfigurability; and
6. closing each individual loop and attaching all compound joints.

The results of this study provide the foundation for future applications where the kinematic performance of reconfigurable polyhedral mechanisms can be exploited.

Available software

There are several software packages currently available for use in the design of origami and origami-inspired devices. A suite of functions written in MATLAB® has been made available to assist in the design of rigid origami structures. This toolbox allows the analysis of Miura-ori (Section 3.1) variations, which are currently the most commonly used crease patterns in engineering applications. *TreeMaker* allows users to generate crease patterns to create virtually any origami base. Similarly, *Origamizer* is a software that generates the necessary crease pattern to fold any polyhedron. A design software called *Freeform Origami* allows crease patterns of a model to be altered and various features of a model, including flat-foldability and developability, to be maintained. *Rigid Origami Simulator* can replicate rigid origami designs given crease patterns as inputs.

A computational origami program called *Eos*, or *E-origami system* has formalized a method for constructing origami models by defining a set of faces and the corresponding fold lines and, although it is capable of mathematical origami folding, it is preferred in artistic origami at this time. Mathematica® also has software packages available that can simulate paper folding and several CAD programs, including SolidWorks®, have the option to use sheet metal as a material, which can be used to test and analyze rigid origami designs.

Conclusions

Origami is an art form that is currently finding many engineering applications. This survey describes the main applications of origami in mechanical engineering. Though it is as yet rare for origami mathematics to be *directly* applied in engineering, the recent expansion of the field has led to algorithms that can be used to define the limits of folding and unfolding, and provide the basis for foundational concepts such as rigid-foldability. Applications have been explored in areas such as aerospace, biomedical devices, packaging, storage, manufacturing, robotics, mechanisms, self-folding devices, core structures, and architecture.

Ongoing research in origami engineering is improving folding efficiency in many engineering operations and recent innovations are expanding the future capabilities and usefulness of these devices.

In order for the results of research in this area to be successfully implemented in applications, some progress is needed in the basic sciences. Among these are: (a) improving understanding of folding algorithms to fold increasingly intricate 3D structures in practice; (b) increasing the mechanical efficiency of folding to achieve cost-effective solutions; (c) determining procedures to modify existing and design entirely new crease patterns that allow folding in more effective ways; and (d) formalizing design approaches and methodologies in origami engineering. It is the authors' hope that the present review will encourage and inspire future origami-based mechanical engineering applications and designs.

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