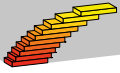


STACK Online Assessment

A collection of case studies

Edited by Malthe Sporning and Chris Sangwin





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Foreword



This booklet contains a number of case studies illustrating the use of the STACK online assessment system.

In 2017 The Open University, The University of Edinburgh and Loughborough University received a National Collaborative Award for Teaching Excellence from the UK Higher Education Academy. These awards celebrate collaborative learning and teaching initiatives that have had a positive effect on the student experience in creative and innovative ways. Each award recognises a team that has enabled a change in practice for colleagues or students at an institutional or disciplinary level. As recipients of this award we wanted to acknowledge more widely, and record for future reference, the contributions made to the STACK project by many colleagues internationally and in a variety of different ways.

These case studies describe how STACK has been used reliably with very large numbers of students at The Open University, and how the international ABACUS consortium have shared learning materials effectively. We record how we have brought online assessment in-house here in Edinburgh, removing the need for external services. A commercial publisher is using STACK to support students learning Physics in schools, and colleagues at Maseno University describe their use of STACK in Kenya.

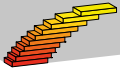
The problems faced by both students and teachers of mathematics, and all STEM subjects, are international. Anyone teaching mathematics with online support will benefit from the functionality STACK offers, and one case study describes how STACK has also been included within the ILIAS learning environment. Students are increasingly learning on their mobile phones, and here we have an additional problem of how to allow students to enter a mathematical expression efficiently without a traditional keyboard. One case study reports how we can benefit from the mechanisms used to enter Japanese hiragana characters, with similar interfaces designed to accept mathematical expressions integrated into STACK.

The nature of education is changing, with students expected to work more independently. We can help these students in ways reported by two case studies, one describing adaptive self-learning exercises, and another the optes project. Lastly, and significantly, we are experimenting with entire courses written online, with STACK questions at the heart of the assessment processes. The case study describing the Fundamentals of Algebra and Calculus course potentially makes the traditional printed textbook redundant.

I am very grateful to all the colleagues and collaborators who have helped develop STACK, and who continue to encourage, to contribute and to improve the system. STACK has proved to be a very useful tool through which we can support students' education. Indeed, STACK (or an equivalent functionality) is now essential to many university departments. This is a very exciting time to be working in education, with rapid changes in technology and the increased availability of technology to more students worldwide. It is gratifying to see how the effort I made in creating and maintaining STACK is proving useful to so many students and colleagues. The STACK community is thriving, and will continue to do so with my support for STACK continuing for the foreseeable future.



Chris Sangwin, The University of Edinburgh, July 2019.



Introduction to STACK

STACK, a System for Teaching and Assessment using a Computer algebra Kernel, is an open source Computer Aided Assessment (CAA) system for mathematics, and other STEM subjects. The first version of STACK was developed in 2004 by Chris Sangwin in collaboration with Laura Naismith at the University of Birmingham. Since its first release, STACK has been continuously developed and is in widespread use particularly in higher education, notably by The University of Edinburgh, The Open University and Loughborough University.

STACK focuses on accepting algebraic input from students, in contrast to traditional online assessment that uses multiple choice questions. Multiple choice questions have a number of well-known limitations. For example, a teacher presents only a small number of answers representing the correct answer and some common mistakes, and students are likely to guess. This can encourage students to focus on tricking the system, rather than solving the problem themselves using mathematics. Mathematics teachers especially need to take care, as here many questions involve “reversible” operations, such as differentiate/integrate and expand/factorise. These can have quite contrasting difficulty levels, and hence in multiple choice questions, students may simply apply the “reverse” operation on all the available options. Multiple choice can normally only assess “lower-order” skills; “Give an example of...” type questions are impossible, despite the research suggesting how beneficial this is to students [3].

Typically a student types an answer which is checked for its “validity”. If the answer is “invalid”, the system will let the student know and give them a chance to rewrite their answer, so students are not punished for lack of understanding on how to type in a particular expression. Once a “valid” expression is accepted, the response is sent to the assessment algorithm.

Give an example of an even function. $f_2(x) =$

Your last answer was interpreted as follows:

$$2 \cdot x$$

The variables found in your answer were: $[x]$

✘ Incorrect answer.
Your answer is not an even function. Look,

$$f(x) - f(-x) = 4 \cdot x \neq 0.$$

Marks for this submission: 0.00/1.00. This submission attracted a penalty of 0.33.

Figure: A typical STACK question. Notice the student’s answer is “interpreted” by the system before they are marked, and that the feedback can reference directly to the student’s answer.

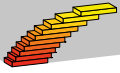
A unique feature of the STACK design separates the inputs and assessment algorithms, and this separation allows multipart questions with many input boxes, and algorithms that mark various combinations of these inputs. For example, this allows follow-through marking, where a student might have given a wrong expression in part (a), but then used their expression correctly in part (b). STACK can recognise the work done by the student in a later part, even though their answer is not the “correct” answer they would have obtained had they got the first part right.

STACK is the most advanced open-source assessment system for mathematics. Open source projects make it relatively easy to collaborate with other teams. Indeed, developing an entire learning system from scratch would be too large a project and commercial learning systems are usually impossible to modify. Ultimately we chose the Computer Algebra System (CAS) MAXIMA to do calculations for STACK. STACK uses other open source programs, e.g. the Learning Environments Moodle or Ilias, LaTeX for mathematics typesetting and MathJax filters to display mathematics.

Features

STACK has the following key features:

- ❖ **Flexible input.** STACK supports the use of algebraic input. This means students can freely enter an equation or a polynomial, and STACK can then assess the properties of this expression. This enables “Find examples of”-style questions, where there are many correct answers. STACK had very flexible input options, for example options to require minimum numbers of significant figures and SI units. STACK also supports “reasoning by equivalence”, where students have to present an argument line-by-line.
- ❖ **Teachers author their own questions.** STACK gives teachers the freedom to write their own questions, while striving to keep question authoring as simple as possible. Question authors do not have to write code to assess answers, and are instead asked to focus on the properties of students’ answers, such as “algebraically equivalent to the teacher’s answer” or “factorised”.
- ❖ **Randomisation.** STACK can generate questions with random variables, ensuring different students see different variants of a question. Random questions support practice and help to reduce copying of answers.
- ❖ **Specific feedback.** STACK grades answers through algorithms known as “potential response trees”, where teachers can give partial marks and tailored feedback depending on the different mathematical properties of the students’ answers. You can, for example, give partial marks for an answer that is equivalent to the teacher’s answer but not factorised, or show a graph of the student’s answer against the correct answer.
- ❖ **Multipart questions.** STACK supports multipart questions, enabling teachers to write step-by-step questions, which can be helpful for students learning a new topic. The multipart question system supports follow-through marking.
- ❖ **Question testing.** Testing that questions work as intended is a difficult and time-consuming task. STACK automates this process, by allowing authors to write “question tests”, in a similar way to how “unit testing” is handled in programming. Hence, authors can easily confirm that their questions work as intended and can confirm that all random variants are of the same difficulty.



- ❖ **Multilingual content.** STACK builds on Moodle's language package functionality to translate question interfaces. STACK has been translated into many languages, including German, Finnish, Swedish, Portuguese, Hebrew and Japanese, and this list continues to grow. Question authors can also include translations for individual questions, allowing students to choose which language they want to see a question in.
- ❖ **Reporting.** STACK stores data on all attempts to all questions by all students, allowing teachers to extensively analyse how students are engaging with their quiz, which can help teachers to improve the material.
- ❖ **Support for sciences.** STACK is built for both mathematics and sciences, and supports questions involving numerical accuracy, significant figures and SI units.

Additionally, STACK has a number of resources to help question authoring, including comprehensive documentation¹, sample materials² and ABACUS³: a large material bank of tried and tested STACK questions.

Community

A steady community of STACK users has grown. This includes universities, secondary schools, commercial publishers and developers. STACK v3.0 was released as an official Moodle plugin in 2012, and by mid 2019 there were just under 900 registered Moodle sites, with an additional 35 universities using STACK via the ILIAS plugin.

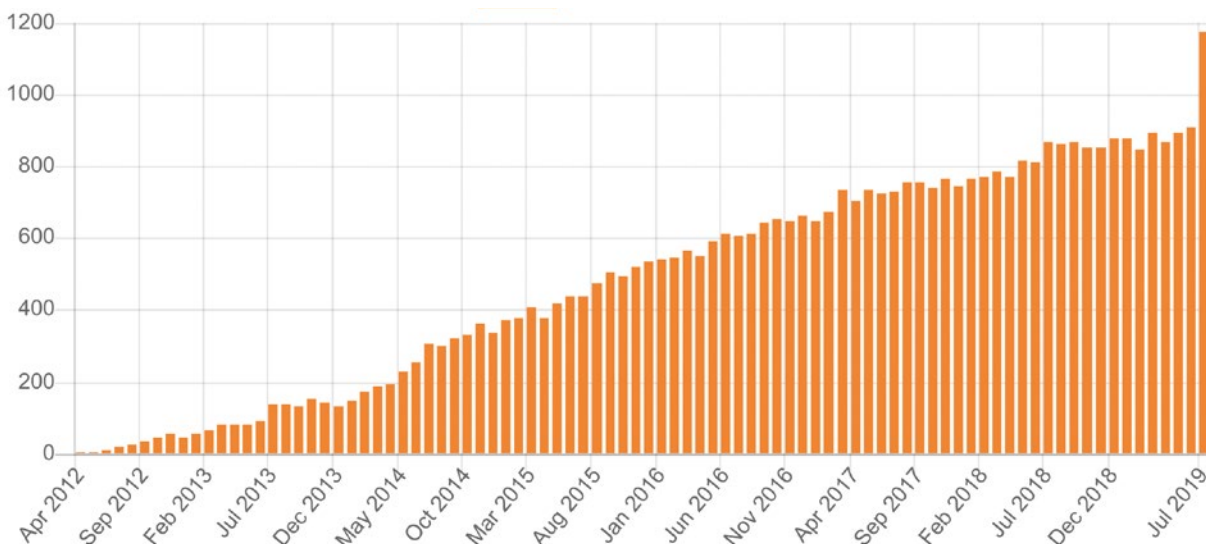


Figure: Number of Moodle sites using STACK over time.

¹ STACK documentation is available locally with any installation, or online at <https://stack2.maths.ed.ac.uk/demo2018/question/type/stack/doc/doc.php/>

² The STACK sample materials are collected in the STACK demo. See <https://www.ed.ac.uk/math/stack/demo>

³ ABACUS is available at <https://abacus.aalto.fi/>

STACK has a dedicated user base who contribute questions to question banks, translate the interface and make additions to the open source code, which is distributed by GitHub⁴. The Git repository has had a steady stream of contributions since its inception, and issues and feature requests are often being raised and fixed. Furthermore, STACK users value the system's usefulness in educational research, and numerous papers have been written about STACK, ranging from research papers to theses and published books.



Figure: Number of contributions to the STACK GitHub over time.

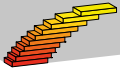
A 2015 report by Chris Sangwin [1], sought to determine in what ways STACK is used by its community. It found that STACK was being used in at least eight languages, mostly for formative and summative quizzes, but also for online examinations and as open access practice sites. The extent of use was very varied, some using it for groups of 20-90 students, most for groups of 100-250 and a small number for 250-1500 students.

The following case studies will provide an overview of how different institutions use STACK, focusing on some key examples of unique and innovative STACK usage. It will provide information on motivations for using STACK, how projects are executed, as well as the barriers and enablers that are involved.

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⁴ The STACK GitHub can be found at https://github.com/math/moodle-qtype_stack



Developing a Fully Online Course

University of Edinburgh, UK

Interview with George Kinnear and Richard Gratwick



THE UNIVERSITY
of EDINBURGH

Abstract

STACK is used in the online course “Fundamentals of Algebra and Calculus”. The course is designed to prepare students for Higher Education (HE) study at The University of Edinburgh, where incoming students have a wide range of mathematical backgrounds. The main barriers were the risk of students using online answer engines to answer questions, and the potential lack of student community. The organizers attempted to address these by clever question design and creating “autonomous learning groups”. The course was enabled by university support, both in the form of a dedicated learning-technologist post and assistance from colleagues. Subsequent diagnostic test results indicate this course was successful.

Introduction

Approximately six hundred students typically study first-year mathematics courses at the University of Edinburgh. They have a wide range of mathematical backgrounds and prior attainments. “Fundamentals of Algebra and Calculus” (FAC) was introduced as an additional course, approximately covering SQA Advanced Higher Mathematics and A-Level Further Mathematics, to address the needs of this diverse group. Since some incoming students do not have the chance to take these higher qualifications before coming to university, the course fulfils a role as part of the University’s Widening Participation strategy, enabling access to mathematics courses for a wider range of students, e.g. students admitted through contextual admissions. This also was an attempt to address a concern from the University that one reason for undergraduate non-continuation was a lack of preparation for the mathematical components of students’ degree programmes, particularly those students on non-mathematics degrees within the College of Science and Engineering. Increasingly, the second-

semester year 1 Calculus and its Applications (CAP) Course had been adapted to address these problems. However, staff and students felt that it recently contained too much material. FAC aimed to relieve some of the pressure on this course.

FAC was delivered as a completely online course to make it scalable, since the above issues are not unique to the School or the University. FAC plays a role in addressing the so-called “mathematics gap” of attainment, preparing students from a wide range of mathematical backgrounds for HE study, and as such could easily see demand from both across the University and beyond.

Why Use STACK?

FAC is essentially a course in mathematical techniques, assessing routine computation. STACK lets teachers assess such questions automatically and give tailored feedback to students. Sophisticated questions can be implemented, for example give-an-example type questions, asking students to provide examples of e.g. a quadratic with given roots, or a sequence with certain monotonicity or boundedness properties. These are important for “retrieval practice”, that is, encouraging students to recall previous topics.

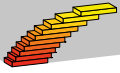
Execution

The course is worth 10 ECTS credits, and consists of approximately 200 hours of work for the student. The course is delivered in ten weekly units, each comprising a number of quizzes for students to work through during that week. Each week ends with a 90-minute “Practice Quiz” the student can take an unlimited number of times, and a 90-minute “Final Test” only allowing one attempt. The test is either given a grade of fail (0-80%), mastery (80-95%) or distinction (95-100%). The high pass mark encourages students to master a topic before moving on. The final grade is determined by combining the results of the 10 “Final Tests” (worth 80%) with a final 2-hour test covering the whole course (worth 20%).

The development involved a substantial up-front time investment: writing the material in these quizzes, including over 900 STACK questions. Although some existing questions were borrowed from other courses, the majority of these questions were authored from scratch. The quizzes interleave textbook-style exposition with videos of worked examples, interactive applets and practice questions, and, as a result, the course material have a polished and varied look. Two lecturers were involved, each responsible for five units, with final cross-checking and occasional extra contributions.

Research

The design of the course was heavily influenced by educational research. Firstly, the course uses “faded worked examples”, that is, presenting a sequence of problems with different amounts of the solution already worked out. For example, a topic might begin with a full worked solution of a problem, followed by a worked solution with the final step as a STACK input, followed by a worked solution with the last two steps as STACK input, and so on. There is evidence that this is helpful for students [2].



Additionally, given the literature on the importance of retrieval practice [3], FAC gives plenty of chances for students to recall what they have learned, for example by having quizzes draw on skills from previous weeks. Weeks alternate between algebra and calculus, spacing out practice of the two topics.

Furthermore, the lecturers decided not to make feedback for the final tests available to students until after the deadline, since there is evidence suggesting delaying feedback is helpful for learning [5].

Finally, FAC gives students a chance to create strong “example spaces” [4], that is, sets of examples that a student can recall for a given topic. Example spaces are developed through give-an-example style questions, which are easy to write in STACK.

Complete the following table of standard derivatives. Try to do this without checking your notes -- which of the most important standard derivatives can you remember?

Function	Derivative
x^n	$n \cdot x^{(n-1)}$ ✓
$\sin(x)$	$\cos(x)$ ✓
$\cos(x)$	$-\sin(x)$ ✓
k	0 ✓
e^x	e^x ✓

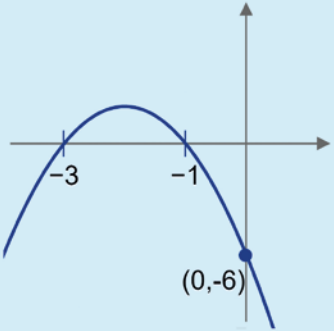
✗ What about the derivatives of these functions?
 $\{\ln(x), \sec(x), \tan(x)\}$

Figure: A FAC question aimed at both retrieval practice and building a strong example space.

Visuals

FAC uses a lot of graphs and interactive applets to make questions visually appealing. Visual intuition is an important part of learning about functions, so it is appropriate for graphical components to play a significant part in this course. FAC uses both JSX graphs (which can be coded directly in STACK) and external plugins, like GeoGebra. The advantage of JSX graphs is that they are self-contained; there is no need to worry about changes to external programs, or licensing rights. There is, however, a cost-benefit analysis to consider. JSX graphs are time-consuming to create for the teacher and have a steep learning curve, while external applets are often quick to set up and have simple drag-and-drop features.

(a) Find the equation of the parabola shown in the diagram.



$y =$

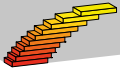
(b) The line with equation $y = k - 8x$ is tangent to the parabola. Find the value of k .

$k =$

Figure: FAC uses graphs and other visuals to aid learning.

Results

The six hundred students taking the first-semester year 1 Introduction to Linear Algebra (ILA) course sat a diagnostic test (also delivered through STACK) in September 2018. In January 2019, the students sat the test again with the same questions but different random variants. In the September test, students who were enrolled on FAC scored on average fifteen percentage points lower than their peers. This is not surprising; FAC will have been recommended to students who felt like they needed more practice in mathematics fundamentals. However, in January, after studying FAC, they had gained these fifteen percentage points and were scoring in line with their peers. This is quantitative evidence that FAC has done exactly what was hoped: removed the discrepancy in attainment between the two groups of students.



The lecturers also wanted to know if taking FAC had improved student performance in other maths courses such as Introduction to Linear Algebra and Calculus and its Applications. Of students scoring similarly in the September diagnostic test, students who took FAC were scoring better than their peers in online quizzes for CAP. They also scored at a similar level to students who did not take FAC in the exams for ILA and CAP.

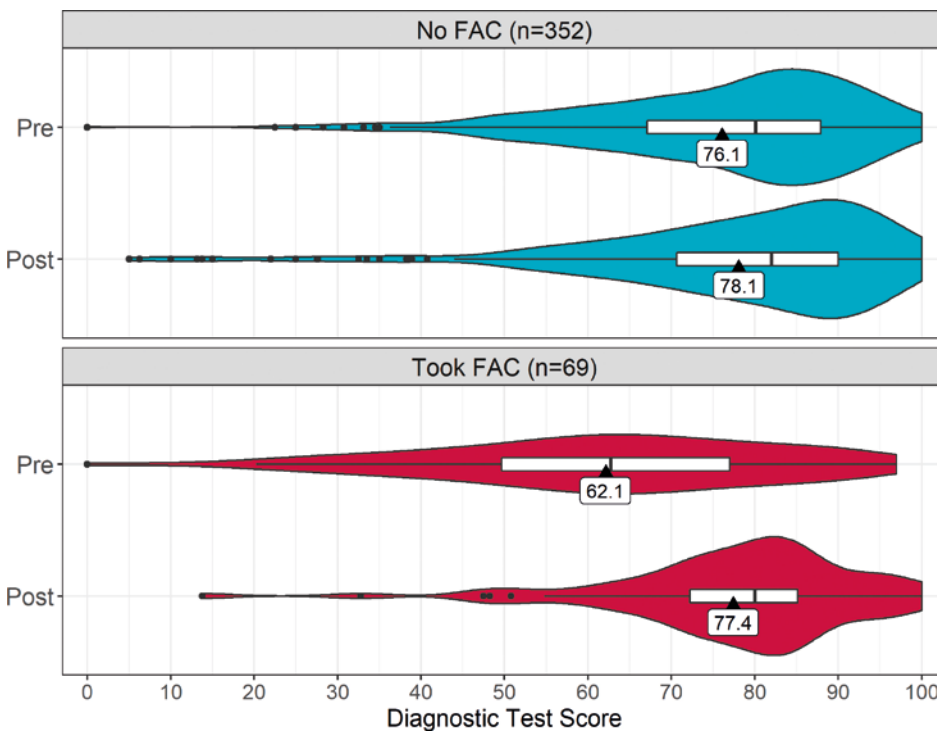


Figure: Diagnostic test results in September (pre) and January (post).

Challenges

While it is true that authoring a simple STACK question is ultimately straightforward, there was nonetheless a technological hurdle to be cleared. One lecturer was entirely new not only to STACK, but to Maxima and any form of online assessment, and therefore was confronted with quite an intimidating prospect. However, they ended up authoring hundreds of questions without trouble, and are now confident in undertaking significantly more demanding questions. This hurdle was easier to overcome since STACK requires no programming expertise.

The lecturers were mindful of the possibility that students could be working through assessed quizzes in one browser tab, with a tool like WolframAlpha open in another. While ultimately this does no benefit to the student, and it remains their responsibility to ensure submitted answers are their own work, the potential for this kind of abuse should be minimized. This just required a bit of cunning at the level of question-setting: e.g. turning “find the derivative of this given function at this given point” into “at what point is the value of the derivative of this given function this given number?”

The limitations of an online course include the difficulties of forming meaningful staff-student and student-student relationships. The lack of real staff-student interaction made it hard to gauge whether the level and volume of material was suitable – anecdotal feedback indicates this was misjudged in at least one of the weekly units. Student-student interaction is important to foster a sense of community. This was addressed by setting up “autonomous learning groups” in which the students could study, in person, once a week, without staff input. Furthermore, students could ask questions in the online forum Piazza, and get in-person help in the MathsBase study room if needed.

Enablers

The University had created a dedicated learning-technologist post to help design the online assessment and write STACK questions. Their work was invaluable for authoring some of the questions, often given only a one-line sketch of the desired question.

The capacity in STACK to clone questions and export questions from other courses was also very useful, making it possible to produce multiple related questions, sharing, for example, a grading structure, without duplicating any work.

Finally, it was an advantage that there is substantial “in-house” support at the University, since STACK is now used across the entire first-year mathematics curriculum and, increasingly, beyond. Hence, there are a number of members of staff with experience of authoring questions, and the students are familiar with it.

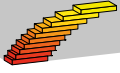
What’s Next?

A lot of data have been collected in FAC’s first year. Quiz responses will be analyzed to determine if any questions should be modified. There are also plans to have interviews with students on the effectiveness of the course, which will help identify areas that need work.

The course organizer will be looking at improving the “autonomous learning groups”. Since groups were given weekly tasks and asked to upload solutions to a shared dropbox, the lecturers can see how much students engaged with these groups. Initially, the engagement was very strong, but half-way through the course, a lot of groups seemed to stop meeting. The course organizer will look at ways to improve turnout for these groups.

The course continues to be worked on. The lecturers are considering options for more interactive questions, for example using equivalence reasoning to assess line-by-line arguments, or interactive JSX graphs that ask a student to, for example, “drag a vector so it is perpendicular to another vector”.

The success of FAC paves the way for similar methods to be used in different courses. Other schools have shown interest in creating similar online courses, and some students may find it helpful to have an online course teaching even less advanced maths.



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STACK for a Physics Textbook

Physics Curriculum & Instruction

David Hilsen

Abstract

The commercial textbook publisher, Physics Curriculum & Instruction, have developed STACK questions to accompany a Physics textbook [1]. These make significant use of random variants and intermediate problem-solving steps. It was important to have feedback on problem design from academic colleagues, and to have support for significant figures and scientific units in STACK. The question bank will contain 3500 questions by the start of the 2019/20 school year.

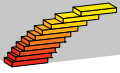
Motivation

Physics Curriculum & Instruction is a Minnesota-based company providing schools with learning resources for Physics. With a lot of open-source options becoming available, putting resources into developing a commercial textbook can carry a significant risk. Physics Curriculum & Instruction hopes to address this by directing their development efforts towards online physics educational resources. It was important to have a system that allowed them to address the wide variety of needs of instructors and that utilised randomization of numerical values. This makes it nearly impossible to search online for the answer to a particular problem; a major concern for publishers.

Execution

A large collection of online STACK questions were developed to accompany the book *Physics Fundamentals* [1]. Access to the online homework package can be purchased along with the book for an extra fee. The online questions are managed in the online learning environment Physics LE, which runs on Moodle. Moodle was picked for its *Enrolment key* feature and its *Adaptive mode* question behaviour that allows students to make multiple question attempts with immediate feedback.

The team had four people authoring STACK questions, and two additional people working on Moodle and the cloud servers. Authoring questions required an average of just over one hour per question with a significant portion of that time devoted to quality control and testing. Ultimately the development sums to a significant financial investment, but also one that leaves an end product of high quality.

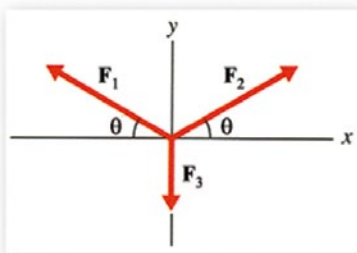


A significant focus in the authoring of questions was on intermediate problem-solving steps, and which ones would be most beneficial to help the student formulate a problem-solving strategy. It was important to go beyond a system in which students simply enter a final numerical value that is marked right or wrong with no additional feedback. The intermediate problem-solving steps used are of the following type:

- ❖ multiple-choice questions, using text or diagrams, checking for understanding and problem-solving approach,
- ❖ input of relevant equations or algebraic expressions,
- ❖ input of intermediate numerical values which need to be calculated to obtain the final answer.

Addition by Components **

Find the resultant of the vectors shown in the figure below. The forces are $F_1 = F_2 = 25.0$ N, and $F_3 = 12.0$ N, and the angle $\theta = 30.0^\circ$.



Part 1 +

Find the sum of the x components for all the vectors.

Find the sum of the y components for all the vectors.

Part 2

Find the resultant of the vectors.

magnitude:

direction: degrees above the positive x-axis

Submit Answer

Figure: A Physics Curriculum question involving intermediate problem-solving steps.

Results

The service went live in January 2019. Since then, 28 schools, both high schools and colleges, have adopted the Physics LE online learning environment. Instructors gave positive feedback, especially appreciating the multiple-part question structure where students are given specific feedback.

Barriers

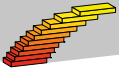
Initially, there was no provision in STACK for handling significant figures, and limited support for physical units. This was a big barrier for Physics question authoring, and hence a more robust system was developed for this purpose. Additionally, high school students sometimes struggle with the syntax for equation input, especially those with minimal computer background. This continues to improve within STACK and may not be a concern in the future.

Enablers

Instructors and colleagues provided feedback from an outside perspective indicating where students might have difficulty. This encouraged the authoring of additional tests, for example checking for a particular misconception.

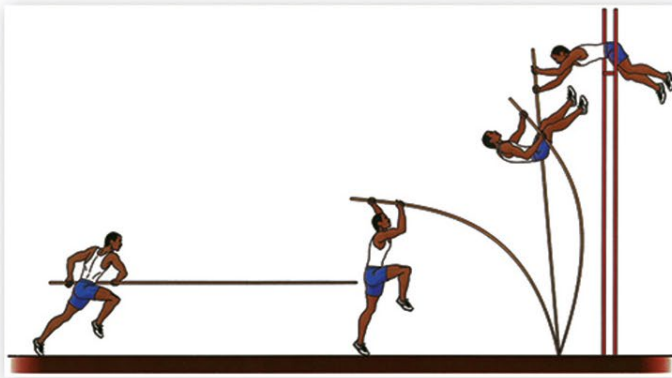
What's Next?

There are many additional schools evaluating the Physics Curriculum online homework system, and additional schools are expected to join. Furthermore, the question bank continues to grow. By the start of the 2019/20 school year, Physics Curriculum plans to have 3500 STACK problems in their system, many of these multiple-part. There are also plans to partner up with OpenStax, a company producing high quality College Physics textbooks, to bring them an affordable online homework solution as well.



Spring Potential Energy; Conservation of Energy **

A pole-vaulter begins a jump with a running start. He then plants one end of the pole and rotates his body about the other end, thereby rising upward. At the beginning of the vault the pole bends, storing potential energy somewhat in the manner of a compressed spring. Near the top of the arc, the pole unbends, releasing its potential energy and pushing the pole-vaulter higher. With what speed must the pole-vaulter approach the bar if he is to raise his center of mass 5.00 m? Assume that mechanical energy is conserved.



Part 1 +

Give an algebraic expression for finding the speed the pole-vaulter must achieve in terms of g and the max height attained (h).

$v =$

Part 2

With what speed must the pole-vaulter approach the bar if he is to raise his center of mass 5.00 m?

Submit Answer

Figure: A Physics Curriculum question involving potential energy.

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[1] V. P. Coletta. Physics Fundamentals. Physics Curriculum and Instruction Inc., Lakeville, Minnesota, 2nd edition, 2010.

Adaptive Self-learning Exercises

Ruhr-Universität Bochum

Michael Kallweit

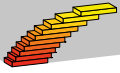
Abstract

At the Ruhr-Universität Bochum, students from all disciplines are offered digital mathematics tasks in an e-learning course for self-study in order to prepare and revise. These questions make intensive use of the randomisation and differentiated feedback options of the STACK question type. Some tasks have been supplemented by adaptive exercises, which guide the students through complex tasks. These adaptive tasks have had great success, are shown to have the same level of student engagement as paper-based tasks and be a better predictor of exam performance.

Introduction

At the Ruhr University Bochum, digital exercises have been used in courses at the Faculty of Mathematics since 2011, and since 2013 this has included STACK. In 2015, a cross-curricular e-learning course was set up, in which students can repeat and process selected topics for exam preparation by themselves. The course takes advantages of Moodle's gamification features: students gain points for engaging with quizzes, contributing to a "level". They also earn badges for completing material in a short time or with a high mark. Additionally, students have access to digital tutorial support.

In addition to these features, the University wanted to add learning exercises that adapt to students' needs.



Motivation

In online assessment, weekly digital exercises are often done in “examination mode”, that is, where students complete tasks in a given timeframe without interacting with the teachers in the meantime. Normally students are only allowed one attempt at the question, and differentiated feedback is only available for the students after the deadline. However, experience shows that students rarely use the detailed written feedback provided and are satisfied with just seeing their score. This observation is consistent with the students’ responses to manually graded paper-based homework.

Additionally, often valuable reflective loops that promote learning are omitted. As highlighted by Hattie’s meta-analysis [2,3], it is important to increase students’ interactivity with the feedback on their solutions. According to [4], good feedback should show concrete possibilities of how gaps in skills can be closed by students.

Contrast this with tutorial lessons, where teachers often help students find a solution path. In the form of minimal assistance, teachers will intervene to help students achieve steady progress in dealing with a problem. If the students take an erroneous path to a solution, the teacher not just corrects the error, but will instead take the student on the path through the task. After being steered on the right path, the student will often fix the error by themselves. This procedure is known as instructional scaffolding: with just minimal support from the teacher, the students can master the individual tasks themselves, and can actively reflect on difficulties and mistakes during the process to close gaps in their knowledge.

The developers at Ruhr-Universität Bochum wanted to bring this concept to online assessment: to build an online assessment system that can carry out instructional scaffolding to help students find a solution path. STACK was ideal for this, as its features in providing specific feedback depending on the properties of students’ answers provide a great foundation for an adaptive system.

Adaptive self-learning approach

Instead of being confronted by a teacher in a tutorial group, students can process adaptive digital self-learning tasks. Following the above principles, students follow an adaptive path of intermediate step tasks in small steps after submitting an incorrectly answered task. The intermediate steps focus on concrete knowledge and skills that must be combined to solve the original task.

The adaptive methods were tested in entrance exams at secondary level II schools, through diagnostic error analysis tools [5,6,7]. The goal of this was to isolate the underlying errors behind why students incorrectly process digital tasks. Using digital tasks with STACK and a complex composition of digital tasks, the researchers achieved a considerable error detection rate of about 90%.

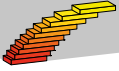
Development of adaptive self-learning tasks

Adaptive self-learning tasks follow the scaffolding principle. The error analysis and subsequent presentation of intermediate steps take place within the potential response trees of a STACK question, and offer added value compared to traditional digital tasks:

- ❖ In contrast to tasks in “examination mode”, students actively deal with difficulties and errors. These are independently overcome by support in small steps and aim at a lasting learning success and increased motivation.
- ❖ Even with randomized STACK tasks, intermediate exercises refer to the task set at the beginning. A reference to the initial problem remains visible along the entire learning path.
- ❖ There are no limits on the design and variety of adaptive paths through the task. With each intermediate step, the next step can be selected on the basis of the student’s individual input.
- ❖ The individual paths through the task are included in the source code of the individual STACK task, which allows questions to be easily shared between courses.
- ❖ The STACK Response Analysis Tool gives teachers insight into the paths taken by students.

Technical implementation

Moodle currently only offers limited possibilities for adaptively designing a sequence of tasks in tests. Hence, the adaptive tasks were introduced at the level of individual STACK questions. For each question, the potential response tree analyses the student’s solution, and if they have made an error, shows a link to the starting point of an individual path. By including an externally stored JavaScript file, the next intermediate step exercise becomes visible only by clicking on a button. The analysis of the input for this intermediate step is also stored in a response tree which adaptively defines the subsequent tasks. By repeating steps and integrating additional ones, the adaptive procedure can simulate real tutorial support.



Example

The following elementary example for matrix multiplication illustrates the central principles of the adaptive task format:

Gegeben seien die folgenden beiden Matrizen A und B mit

$$\mathbf{A} = \begin{bmatrix} -4 & 2 & 4 \\ -4 & 4 & -1 \\ -4 & 0 & 4 \end{bmatrix}$$

und

$$\mathbf{B} = \begin{bmatrix} 2 & 2 & -3 \\ -2 & -3 & -1 \\ -2 & -3 & -2 \end{bmatrix}$$

Berechnen Sie das Produkt $C = A \cdot B$ der beiden Matrizen:

$$\mathbf{C} = \mathbf{A} \cdot \mathbf{B} = \begin{bmatrix} \boxed{} & \boxed{} & \boxed{} \\ \boxed{} & \boxed{} & \boxed{} \\ \boxed{} & \boxed{} & \boxed{} \end{bmatrix}$$

Figure: An adaptable question about matrix multiplication.

If the input of the solution is incorrect, the student is informed that they can work on the task again in guided intermediate steps via the “Weiter” (“Next”) button.

Das ist leider die falsche Antwort.

Die rot unterstrichenen Einträge sind falsch.

$$\begin{bmatrix} -20 & \underline{26} & 2 \\ \underline{14} & -17 & \underline{-10} \\ -16 & \underline{20} & \underline{-4} \end{bmatrix}$$

Bitte beginnen Sie die Bearbeitung der Aufgabe in drei angeleiteten Zwischenschritten noch einmal.

Klicken Sie auf den Button, um den ersten Zwischenschritt anzuzeigen.

Weiter

Figure: In the feedback, students may be told to work through the problem again, step-by-step.

The first intermediate step then opens directly below the actual task, in this case asking students to identify the rule that is used when calculating matrix products. Subsequent steps will be available each time a step is correctly completed and the student clicks “Next”

Zwischenschritt 1A: Mit welcher Rechenregel wird das Matrixprodukt berechnet?

- Komponentenweise Multiplikation
- Spalte (von A) mal Zeile (von B)
- Zeile (von A) mal Spalte (von B)

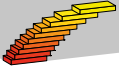
Zwischenschritt 2A: Berechnen Sie den Eintrag C_{11} der Matrix C .

$$C_{11} = \boxed{}$$

Zwischenschritt 2B: Um den Eintrag C_{11} der Matrix C zu berechnen, muss die erste Zeile von A komponentenweise mit der ersten Spalte von B multipliziert und aufsummiert werden. Berechnen Sie daher

$$(1) \cdot (4) + (1) \cdot (1) + (-3) \cdot (1) = \boxed{}$$

Figure: The three intermediate problem-solving steps for this question about matrix multiplication.



The entire adaptive path with all intermediate step tasks has the following structure.

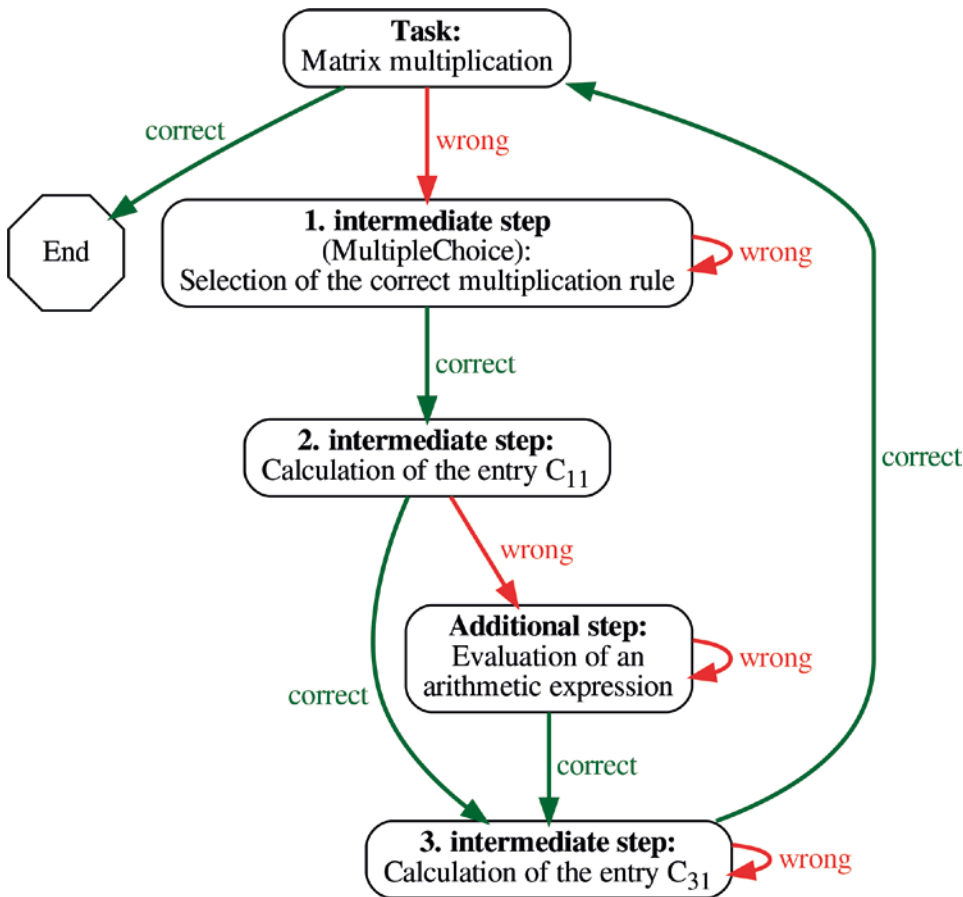


Figure: An overview of the adaptive path for the matrix multiplication question.

It is also possible to ask students to make a decision about which direction they want to take.

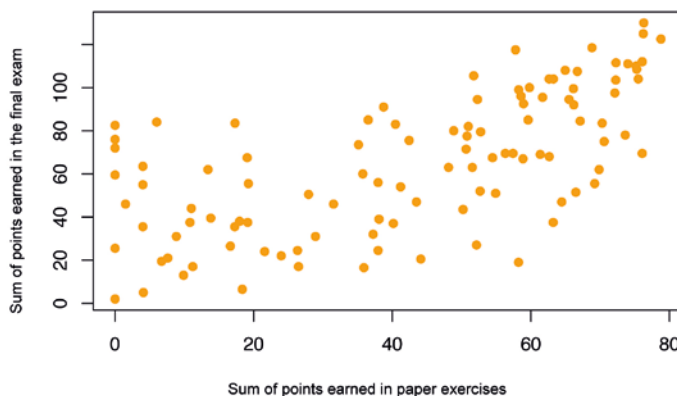
Branches at which students make decisions about which direction they want to take are also possible. Overall, the feedback can be structured in a variety of ways according to Zech's hierarchy levels, with a focus on motivation, strategy or content help [8].

Results

When comparing the University's new online assessment to their old, multiple-choice based system, several improvements become apparent. In multiple choice questions, you can only have a finite number of "distractors", but in STACK you can give feedback on an unlimited number of answers. In particular, the developers compared a question about expanding $(2x+3y)^2$, and found that while the old system had only 9 distractor options, the STACK system had registered 41 different answers over its lifetime, which could be grouped into 28 different types of mistakes. This is a major improvement on the number of mistakes students can receive specific feedback about.

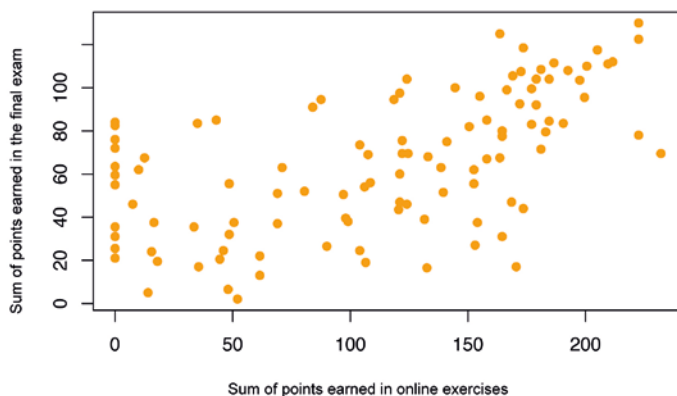
Additionally, the developers wanted to compare the performance of paper-based and digital homework. Across all courses, they did not find any significant difference in how many tasks students complete between the two systems. This is encouraging; the University saves resources by not relying on human markers for homework, and this confirms that they have not lost any functionality or student engagement by moving towards online assessment. For the course "Mathematics for Chemistry", the correlation between paper-based homework scores and exam scores was found to be 0.56, but the correlation between online homework scores and exam scores was 0.63. Similar results were found for the other courses at the University. Hence, online quiz scores are a better predictor for exam scores.

Correlations



Weekly paper exercises
vs. final exam:

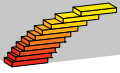
$$\rho=0.58 \quad (p<0.00001)$$



Weekly online exercises
vs. final exam:

$$\rho=0.65 \quad (p<0.00001)$$

Figure: Comparing both paper-based and online homework scores with exam scores, for the course "Mathematics for Chemistry".



Challenges

One of the main challenges of STACK integration was keeping up with new versions. It was important to convince administrators that going through the trouble of updating STACK was worth it, as new updates and features could be important to question authors.

Additionally, there was a challenge in distributing the questions to new users, since Moodle's options for managing and distributing questions were found lacking. To solve this, a database for managing STACK questions in moodleXML files was created, which included more sophisticated search options. For each question, the system generates an overview pdf file with a short description, information on available languages, screenshots and an automatically generated overview of the potential response trees. About 700 questions are in the database, and because of its usability, it is easy to convince new users that it is simple to find good STACK questions.

Enablers

It was helpful to have a strong community of STACK users to help with projects. For example, one of the lecturers held a seminar to teach effective STACK usage to students training to become school teachers. The students of this class became a valuable community of STACK users willing to help with other projects. Additionally, it was valuable that STACK had been translated to German, as this allowed the University to easily spread STACK to other German institutions and build good partners.

What's Next?

The STACK courses at Ruhr-Universität Bochum continue to be improved on. Future additions may include using more sophisticated features in questions, such as equivalence reasoning inputs and interactive JSXGraph visuals.

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A Flick Interface for Maths Input

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Yasuyuki Nakamura

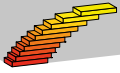
Takahiro Nakahara (Songensha LLC.)

Abstract

Typing mathematical expressions into mobile devices can be time-consuming. To solve this problem, developers at Nagoya University are developing a “flick interface” for STACK, similar to the popular Japanese keyboard mode. Students can pick a common integer or variable and flick in a direction to quickly turn, for example, x into \sqrt{x} . A survey of usability suggests students prefer this input type to their traditional keyboard.

Motivation

As smartphones become increasingly used in schools and universities, it is important to find a reliable way to input mathematical expressions. Inputting mathematics on a computer is not a problem, but on smartphones and other mobile devices it can be much more time-consuming. For example, to type an answer to the question “Expand $(x+2)(x+3)$ ”, students have to enter the expression x^2+5x+6 into the answer space. On a mobile device, this requires several switches between the alphabetical and numerical views of the keyboard, resulting in 19 key touches for this simple answer.



Recently, a flick keyboard for Japanese characters has become very popular in Japan, especially amongst young students. The keyboard shows only a limited number of hiragana¹ characters, but students can hold and “flick” on a key to choose a similar character. Characters are grouped together by sound, for example grouping together na, ni, nu, ne and no. The keyboard will then suggest related kanji and/or katakana characters that share similar sounds. It was the popularity of this keyboard that motivated the developers to create a similar interface for inputting mathematics, as a STACK input type.

Execution

The flick interface was developed by Yasuyuki Nakamura and Takahiro Nakahara, funded by a grant from the Japan Society for the Promotion of Science. Like the Japanese character keyboard, students are faced with a small set of common inputs. Students can switch between numerical values and common variables by pressing “123” and “xy”, respectively, or see a full keyboard view by clicking the keyboard logo. Pressing on a number reveals some common manipulations, and students can then flick in one direction to turn, for example, “2” into “2x”. The input is compatible with normal keyboards, and will automatically be selected for the type of device being used.

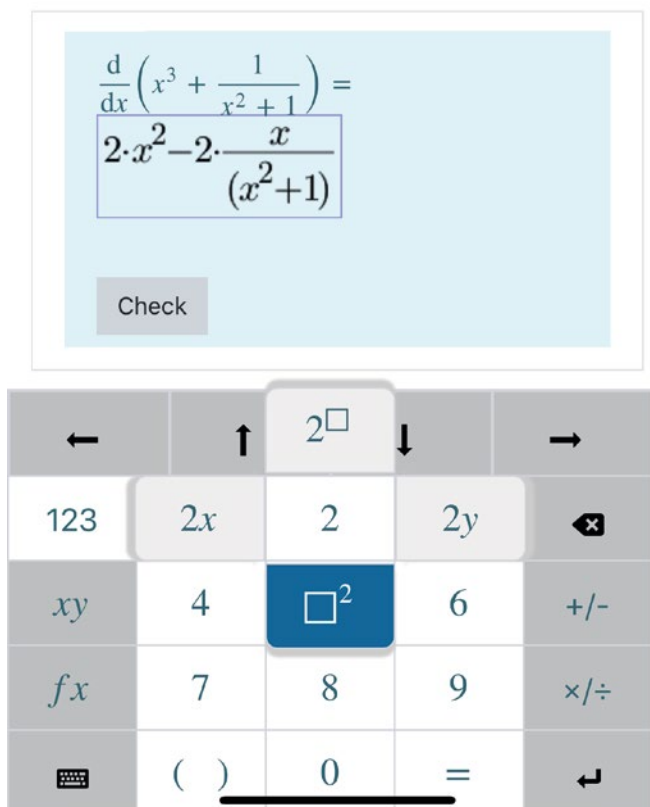


Figure: The flick interface in action.

¹ Japanese has three character systems. Hiragana is mostly used for simple and native Japanese words, katakana mostly for foreign words and kanji for all other words.

The interface is programmed in JavaScript to minimise the dependency on mobile device operating systems. The interface uses MathDox² to describe expressions, and the input is then converted to the Maxima format by a conversion filter previously created by the developers. Developing an interface suitable for all different screen sizes was one of the biggest challenges of the project, but now the interface is suitable for most devices of sizes around 5-6 inches.

Results

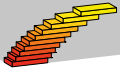
The flick interface is estimated to significantly reduce the number of keypresses required to type an expression. The following table shows an estimated comparison for a few common expressions [1].

Mathematical expression	Direct input taps	Flick input taps
x^2+5x+6	19	8
$3x^2 - \frac{2x}{(x^2+1)^2}$	36	13
$2x \cos(x^2)$	23	7

To analyse the usability of the interface, the developers conducted a usability test at Nagoya University, Japan. 29 students were asked to enter five common maths expressions by using both a traditional keyboard and the new flick input keyboard. After they entered these math expressions, they were given a survey on usability and satisfaction levels. The survey questions were based on the following five parameters, originally from Jakob Nielsen's five goals of usability [2]: learnability, efficiency, difficulty or ease in making corrections, rememberability, and the intent to reuse. For each parameter, students were asked to give each input type a score from 0.0 to 5.0. The following table shows the results [1], including both the averages, as well as the standard deviations in parentheses. The survey suggests that the usability and satisfaction levels are higher when the flick input method is used to enter mathematical expressions, rather than the direct input method.

Question	Direct input score	Flick input score
It is easy to learn how to input math	3.0 (1.2)	3.5 (1.0)
I can input math quickly and easily	2.6 (1.1)	3.2 (1.3)
It is not confusing and easy to correct	3.0 (1.2)	3.1 (1.1)
I remember the method that I learnt at the rehearsal	3.0 (1.1)	3.1 (1.1)
I will use this method to input math the next time	2.9 (1.3)	3.2 (1.4)

² See <http://mathdox.org>



What's Next?

The next step for the developers is to merge the flick interface into STACK as a new input type. This will include refactoring the flick interface code and working with STACK developers to best implement it into the question type.

References

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Innovating Education in Maseno, Kenya

Interview with Santiago Borio, IDEMS and Michael Obiero Oyengo, Maseno University

Abstract

IDEMS International is working with Maseno University to implement online assessment for their mathematics courses. Developers from IDEMS and Maseno University created online quizzes for two courses during the pilot of the project, running on servers at the University of Edinburgh. The main challenges of the project were related to poor student access to WiFi and time pressure for the question authors. The feedback from the students was mostly positive, and the pilot paves the way for more similar work in the future. This includes online assessment for many more courses at Maseno University, as well as collaborations with other African universities and secondary schools.

Introduction

IDEMS International (Innovations on Development, Education and the Mathematical Sciences) is a Community Interest Company working to support different development causes, mostly in Africa, in fields of education and mathematical sciences. The non-profit organisation generates open education resources and offers services to Universities and other institutions. One of their projects was to help implement Computer Aided Assessment (CAA) in African Universities, of which Maseno University was chosen as a pilot.

Maseno University, Kenya, has a large number of students in its undergraduate mathematics courses, many courses having 800 enrolled students and some more than 1000, but there is little support for lecturers in terms of marking. This puts pressure on their ability to reliably do weekly assessments for students. By changing the continuous assessment from a paper format to an electronic system, the hope was that lecturers would have more opportunities to concentrate on their teaching and supporting students in other ways.

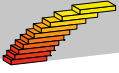


Figure: Maseno University.

Why Use STACK?

There were a number of reasons for why STACK was chosen for this project. Firstly, there was already some familiarity with the STACK system within IDEMS, as one of the contributors had previously worked with STACK question authoring. Secondly, the main STACK developer offered assistance, including the opportunity to run the service on servers at the University of Edinburgh. Finally, having compared STACK with other systems, the developers found that the flexibility in answer tests, recognition of mistakes through potential response trees and the ability to have questions that require numerical or algebraic input for answers made STACK more powerful than the alternatives.

Execution

The developers from IDEMS and Maseno University created online STACK quizzes for the courses “Calculus I” and “Introduction to Linear Algebra”. In the case of “Calculus I”, the main lecturer provided a written quiz structure and list of questions, so the main work was just typing these into STACK and implementing randomisation. In contrast, the quizzes of “Introduction to Linear Algebra” were written mostly from scratch, only using a few questions from the STACK sample materials.

Initially, students were given a syntax quiz to help them learn the STACK syntax, and then in each of the following 10 weeks they were given a formative “mastery quiz” and a summative “test quiz”. The mastery quiz could be taken as many times as the students wanted. Students were required to get a score of 80% in the mastery quiz to unlock the single attempt at the respective test quiz. In “Introduction to Linear Algebra”, this restriction was later reduced to 70%, as lecturers felt students were having a hard time keeping up with materials. The mastery and test quizzes together count for 30% of the student’s course grade. Towards the end of the course there was also a review week with STACK questions, and in “Introduction to Linear Algebra” students could take an online practice exam.

Determine the value of the constant A for which the function

$$f(x) = \begin{cases} \frac{x^2-6x}{x-6} & x \neq 6 \\ A & x = 6 \end{cases}$$

is continuous.

$A =$

Check

Figure: A STACK question from “Calculus I” involving continuity and L’Hôpital’s rule.

Feedback

After the first year, the students of the two courses were asked to fill out a survey about their experience with STACK. The first few questions regarded accessing the course. Maseno University does not have reliable WiFi, and it is not common for students’ accommodation to have WiFi either. As a result, most students had to use either their own or a friends’ phone, and purchase a data package to access the course. Indeed, when asked to rate the difficulty of different aspects, many students ranked “accessing the internet” in the range of “slightly challenging” to “very challenging”.

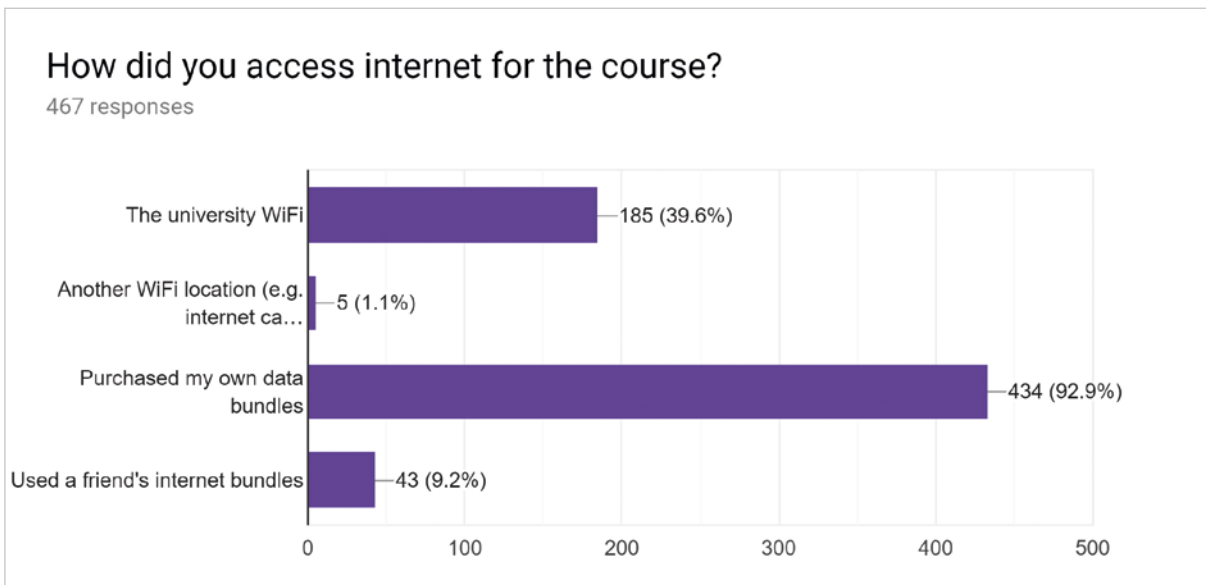
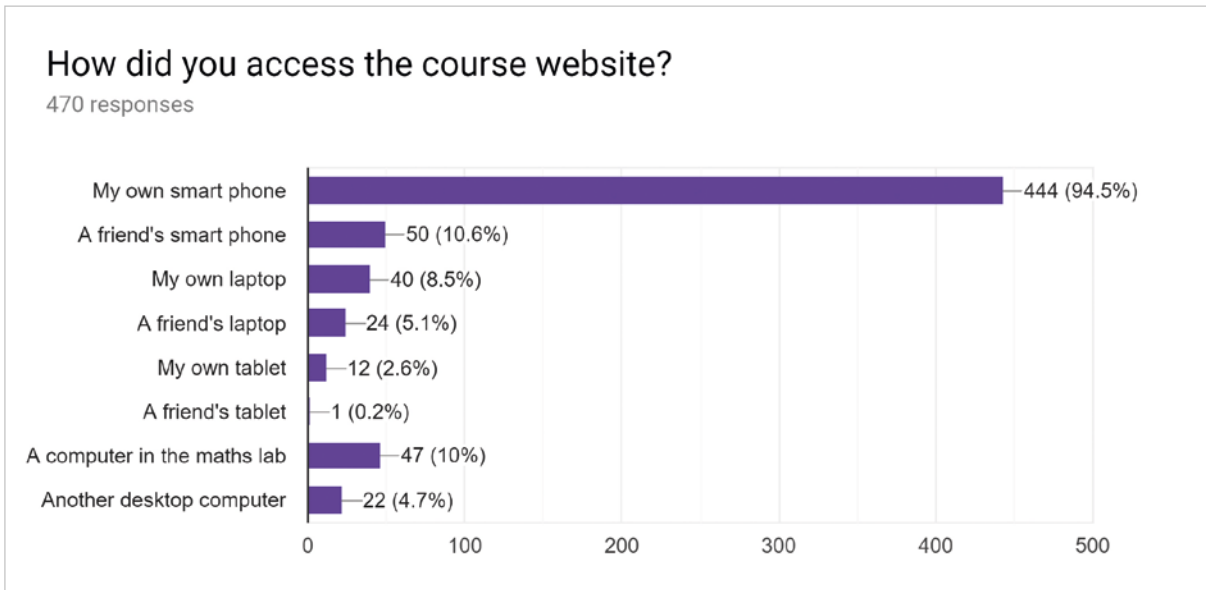
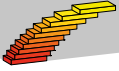


Figure: Responses to the survey for “Introductory Linear Algebra”.

When asked to rate how helpful they found different aspects of the assessments, many students ranked getting feedback, being able to have multiple attempts at mastery quizzes, having weekly quizzes and having the mastery quizzes open all semester as useful. Most students did not seem to have too much difficulty with the STACK syntax, ranking it in a range of “not a challenge” to “moderately challenging”. This was surely helped by the attempts to ensure students were learning the syntax, for example through the introductory syntax quiz. However, it also seems there is room for improvement here.

The students were also given an opportunity to add any general comments or suggestions. The majority of these answers were from students who were happy with STACK and said it was useful for getting a deeper understanding of the material.

With regards to quizzes, many students requested more than one attempt at test quizzes, with extended deadlines. They requested more mastery quizzes, suggesting they found them helpful, and some students suggested access to harder, exam-style questions.

There were also some comments about the feedback for questions. Due to time constraints, the calculus course only had feedback in the form of worked solutions, and for linear algebra, students were only given outlines to solutions. Students said that they wanted better feedback, and so one of the goals for the next iteration will be to add tailored feedback to all questions.

Finally, there was a call to improve accessibility for the visually impaired.

Challenges

As is evident from the survey responses, there was a challenge regarding the students' access to internet. However, this did not end up being as big a problem as anticipated, as students seemed able and willing to pay for data packages to access the course.

Furthermore, due to the short term allocation of courses and lecturers, there was a significant time pressure for writing the quizzes. As a result, there was no time to implement tailored feedback, and a number of quizzes were delivered late. However, this did not seem to discourage students or create major issues in the completion of quizzes.

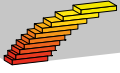
Enablers

A major enabler for the project was the ability of IDEMS to assist the University in question authoring. The questions were written by Michael Obiero Oyengo and David Ambogo from Maseno University and Santiago Borio, Danny Parsons and David Stern from IDEMS. Additionally, the main STACK developer Chris Sangwin helped give crash courses in using STACK, which sped up the learning process. The course was also run on a server at the University of Edinburgh, which meant the developers did not have to worry about setting up a server and could focus entirely on question authoring. Finally, the STACK sample questions¹ helped speed up the question authoring process, as the developers could use them as templates when writing similar questions.

What's Next?

Following the success of this pilot course, IDEMS is looking to expand into similar projects. The first step will be improving the two initial courses. Once the analysis of the data from the pilot course is finished, it will be possible to identify the main areas that need work. These two courses have been transferred to an IDEMS server, so they can be easily used for online assessment at other universities. The server will be built to cope with universities with many students, as IDEMS expects to use it for some courses with more than 1000 students.

¹ See the STACK demo: <https://www.ed.ac.uk/math/stack/demo>



A number of Universities have expressed interest in this project, including universities from Kenya, Ethiopia, Rwanda, Tanzania and Uganda. In particular, the mathematics department at Bahir Dar University, Ethiopia has agreed to investigate implementing a number of IDEMS' courses. In August 2019, IDEMS is holding a workshop where representatives from these universities will discuss STACK and how to best implement e-assessment at their institutions. The 5-day workshop will include talks on the advantages of using online assessment, as well as practical work to prepare lecturers who want to use IDEMS' online course material. The workshop is funded by a grant from the Commission for Developing Countries (CDC) of the International Mathematical Union.

IDEMS is also helping Maseno University create online materials for several of its other courses. In the 19/20 term, Maseno University will run the improved "Calculus I", as well as five other courses with online assessment: "Descriptive Statistics", "Introduction to Probability Theory", "Basic Mathematics", "Calculus II" and "Vector Analysis". For these courses, IDEMS will be training some lecturers to author their own questions, which will increase their control of the online quizzes. As with the two pilot courses, these courses are planned to be piloted at Maseno University first, then refined based on feedback and made available as an OER for other universities.

Finally, IDEMS is looking at using STACK in secondary schools. There are plans to develop an e-book for African schools with accompanying STACK questions, including an offline system for schools with poor WiFi. IDEMS is also evaluating opportunities to develop an open assessment system for English schools.

optes: Optimising Self-study With STACK

DHBW Mannheim

Miriam Weigel, Katja Derr, Reinhold Hübl

Abstract

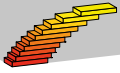
The **optes** project uses STACK in their pre-course, designed to help students improve their self-studying skills for their first year at university. The tests were developed and maintained at DHBW Mannheim. Here, students take a diagnostic test, and based on their results are given access to various learning modules, comprising text, graphs, animations, examples and exercises. An analysis of data gathered from the first evaluation phase of the project shows that the pre-course is effective at bridging gaps in school knowledge: at-risk students who end up scoring highly in the pre-course show improved performance in their first year mathematics courses.

Introduction

Students entering higher education are a diverse group and many students have considerable gaps in school knowledge, putting additional pressure on their first year at university. This particularly applies to mathematics, as basic skills in this subject are considered a prerequisite to successfully complete a course in STEM subjects. As a consequence, many universities provide preparatory courses in mathematics, either face-to-face, web-based, or in blended learning scenarios.

The joint research project **optes**, funded by the German Federal Ministry of Education and Research (BMBF), develops and evaluates learning materials and tools that help students “refresh” their basic knowledge in mathematics and support the development of learning strategies. **optes** stands for “Optimierung der Selbststudiumsphase”, which translates to “optimisation of the self-study phase”.

Over the course of eight years, **optes** developed a comprehensive web-based pre-course consisting of diagnostic tests, learning modules and concepts for web-based student support, and tested it at the participating universities. The partner universities include Baden-Wuerttemberg Cooperative State University (DHBW) Karlsruhe, DHBW Mannheim, DHBW Mosbach, University of Applied Sciences and Arts Ostwestfalen-Lippe, Universität Hamburg; Cooperating universities: Julius-Maximilians-Universität Würzburg, University of Education Heidelberg.



Why Use STACK?

The project put a strong focus on the development of self-tests that enable learners to apply their knowledge and provide them with meaningful feedback. This required a sophisticated CAA system, especially one that allowed students to enter algebraic input. Since **optes** results are published under Creative Commons or General Public licenses, it was also important that the system was open source. To find the most suitable CAA system, the researchers analysed and compared many existing mathematical tools and in 2013, STACK was chosen. One of the reasons for choosing STACK, is that STACK uses the Learning Management System (LMS) Moodle and the Computer Algebra System (CAS) Maxima, both of which are supported by large communities. This gave the researchers confidence that STACK would also be used and developed further. Additionally, STACK is flexible enough to allow researchers to design very different types of mathematical problems. The only barrier was that STACK was not available for the LMS ILIAS at the time, a system widely used in Germany. However a crowd funding initiative was successful in gathering enough funds to implement STACK into ILIAS.

Execution

The partner university DHBW Mannheim was responsible for the development of tests and self-assessments. Here, a very basic version of the pre-course was implemented in 2013, and based on repeated evaluations, the different tests and learning modules were successively built and improved upon [2]. While some learning resources already existed in the form of printed scripts and paper versions of tests, the complete course material had to be rewritten and typed into ILIAS. In 2019, the course held ten interactive learning modules, with more than 1500 mathematics test items, 140 of which use STACK.

Figure 1 shows an overview of the course structure as executed at DHBW Mannheim since 2014. The programme runs in the months leading up to a new semester, and starts with a diagnostic self-test covering the entire pre-course's syllabus.

Depending on their diagnostics test results, students may access the different learning modules. Each course provides text, graphs, animations, examples and exercises, and at the end of each module, students can take a subject-related test consisting of 10 to 15 randomised questions. Students who want additional support can enroll in e-tutored courses, where their learning process is structured and monitored by e-tutors. Students can then discuss problems and test results with peers and e-tutors, and are required to upload completed exercise sheets.

During induction week, all participating first year students take a final test at the University's computer laboratories. Since 2014, more than 2800 students have participated in the diagnostic pre-test and the pre-course at DHBW Mannheim. This corresponds to approximately 560 students per year, which is around 80% of the cohort.

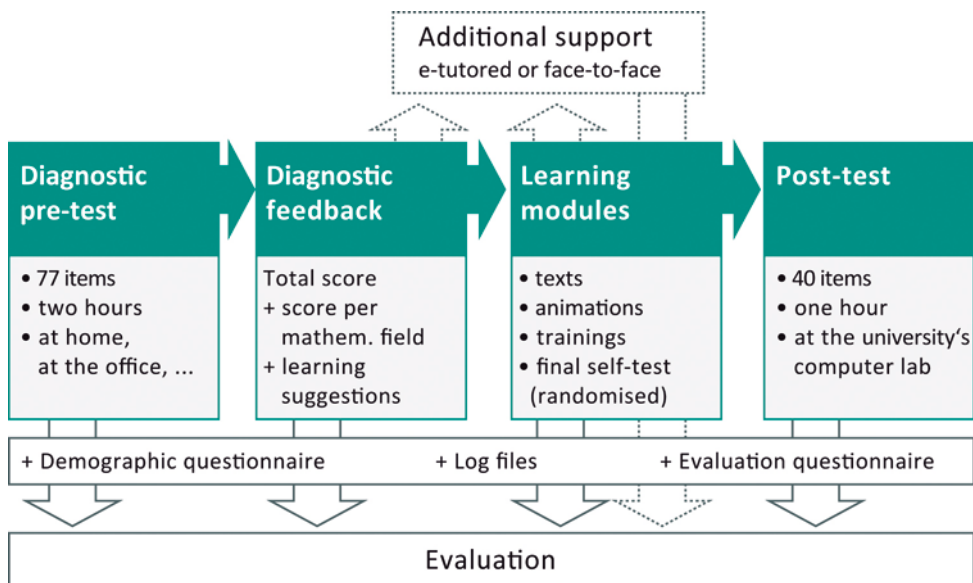


Figure 1: Course design at DHBW Mannheim.

While the diagnostic self-test aims at informing learners of their level of knowledge in relation to the curriculum, the self-tests provided in the ten different courses are designed to encourage learners to independently practice their skills.

Question Construction

Figure 2 shows an example of an **optes** question in ILIAS. Students can click the checkmark next to the input box to have their answers validated by the system. On top of the question, the student can click to pop-out an explanation of the input syntax. The question is graded by clicking on “Rückmeldung anfordern”, which means “Request feedback”.

In this example, the student is asked to give an example of a function with three given zeroes. There are no other constraints, and in particular the function can also have more zeroes. The given zeroes are randomly generated, so the student can restart the test to try a variant of the problem with different constants.

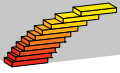
▼ Wichtige Hinweise zur Eingabe-Syntax

Geben Sie eine Funktion $f : \mathbb{R} \rightarrow \mathbb{R}$ mit den Nullstellen $x_1 = 7$, $x_2 = -4$ und $x_3 = -6$ an!

$f(x) =$ ✓

Rückmeldung anfordern

Figure 2: An optes question about giving an example of a function with given zeroes.



optes questions focus on providing good feedback to students [7]. Feedback incorporates partial marks, if the student for example entered a function with only one or two of the given zeroes, and visuals, by for example graphing the student's answer (in red) against a correct answer (in blue), see figure 3. The student is also shown the value of their function at the three given points and given a model solution. **optes** questions are great examples of effective use of STACK's feedback features.

Ihre Antwort ist nicht korrekt.

Ihre Funktion nimmt für $x = 7$ nicht den Wert 0 an: $f(7) = 42$

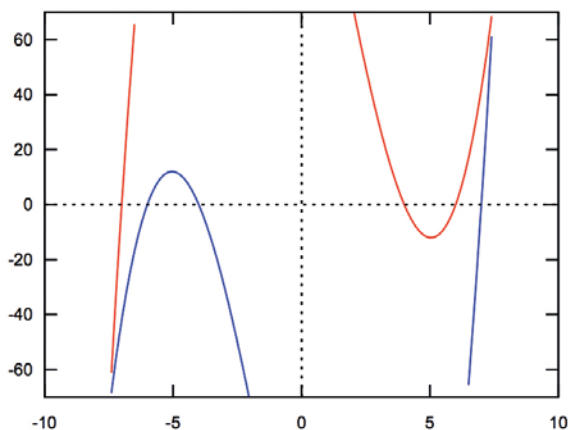
Ihre Funktion nimmt für $x = -4$ nicht den Wert 0 an: $f(-4) = 240$

Ihre Funktion nimmt für $x = -6$ nicht den Wert 0 an: $f(-6) = 120$

Sie haben wahrscheinlich einen Denkfehler bei der Wahl der Vorzeichen gemacht:

Sie haben für die Funktion $(x - 6) \cdot (x - 4) \cdot (x + 7)$ statt $(x - 7) \cdot (x + 4) \cdot (x + 6)$ geschrieben.

Das Schaubild zeigt die **Musterlösung** und **Ihre Antwort**:



Lösung:

Unendlich viele Funktionen erfüllen die Bedingungen aus der Aufgabenstellung.

Eine mögliche Funktion lautet: $f(x) = (x - 7) \cdot (x + 4) \cdot (x + 6)$

Erläuterung:

Wir suchen eine Funktion $f(x)$, die die Bedingung

$$f(7) = f(-4) = f(-6) = 0$$

erfüllt.

Eine Funktion mit diesen Eigenschaften ist beispielsweise:

$$f(x) = (x - 7) \cdot (x + 4) \cdot (x + 6) = x^3 + 3 \cdot x^2 - 46 \cdot x - 168$$

Figure 3: Feedback to the example question shown above.

Results

The major goal of the **optes** project was to improve students' self-study abilities in mathematics related subjects. Using data collected from the first evaluations phase of the project, the researchers studied the influence of taking the pre-course on different variables. They found that the biggest factor in study success was a student's previous knowledge and secondary school scores. However, they also found that the pre-course could be very effective at improving first-year performance. Of the students whose performance were at risk, those who scored highly in the pre-course also did better in their first year of study. These were the students with poor prior study success or little domain-related knowledge. Self-test engagement was also found to have significance, as they were related to both pre-course gains and first-year performance.

Challenges

A major advantage of learning management systems is their ability to provide automated feedback to students. Instructors are hence relieved from the load of marking hundreds of exercises, and students appreciate the immediate response of the system. However, writing feedback for web-based mathematical problems is time-consuming, and these efforts may grow exponentially when the questions have multiple correct answers. To help soften this burden, it is important to share STACK questions, not only across pre-course projects but also across universities and e-learning platforms.

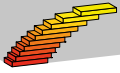
Enablers

A major enabler for the project was the initiative to integrate STACK into ILIAS. If this crowd funding initiative had failed, **optes** may have had to choose a less suited CAA system.

Furthermore, a lot of institutional support enabled this project. While all resources developed by **optes** are Open Educational Resources (OER), the implementation of the learning material at third party universities or educational institutions demanded staff and technical infrastructure to install ILIAS and STACK. Staff and technical support helped administer the pre-course and adapt the learning material to each university's needs. Furthermore, the e-tutor system was enabled by lecturers and older students who took the time to learn to use STACK so they could help pre-course participants as e-tutors.

What's Next?

Since 2019, the rollout of **optes** to other universities and institutions has begun, and as more and more lecturers use the **optes** resources, the STACK community in Germany continues to grow. This community is largely represented by the working group "Mathe digital". Lecturers are encouraged to develop and contribute their own questions, resulting in a growing and improving database of available STACK questions.



The researchers continue to evaluate the data from the project. When this analysis is finished, it will be possible to better identify the strengths and weaknesses of the course structure.

Like all projects funded by the German BMBF programme “Quality pact for teaching”, **optes** will be finished by the end of 2020. The developed concepts and course material, however, will be used and developed further at the **optes** partner universities and at all universities and institutions that share the material. Future plans include incorporating adaptive testing and expanding the work to other subjects and topics, such as mathematics for business and psychology courses.

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Technical Integration of STACK into ILIAS

Innovation in Learning Institute

Interview with Fred Neumann and Jesus Copado



Abstract

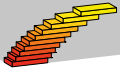
STACK was integrated into the ILIAS learning management system to support projects for learning content. This was done by creating an integrated question type directly in ILIAS. The biggest challenges were decoupling the dependencies on Moodle, and an increasing number of user queries following a quickly growing community.

Motivation

ILIAS¹ is one of the most popular Learning Management Systems (LMS) used in German universities. It has been available under an open-source license since 2000, and since then has grown in popularity in a number of countries, especially Germany and Switzerland. In the last decade, the ILIAS community started a special-interest group related to mathematical assessment. The main driver for this was the 'optes' project, a federally funded project to create open learning content for mathematics and natural science. It was key for this project to have a sophisticated question system that uses CAS to evaluate students' input. In 2013, the group compared different systems for CAS based questions and finally decided to use STACK.

STACK was initially built for the learning management system Moodle, and it would not be trivial to implement STACK into ILIAS. A crowd funding initiative was started to port the STACK plugin from Moodle to ILIAS, and the Innovation in Learning Institute at the Friedrich-Alexander-University Erlangen-Nuremberg (FAU) was asked to implement and maintain it. This would also be an advantage to FAU, as it could then establish STACK as an innovative element of e-learning and e-assessment at the University.

¹ ILIAS homepage: www.ilias.de



Execution

It was important to create an integrated question type that works like all the others in ILIAS, instead of trying to dynamically connect a separate STACK platform to ILIAS. Initially, a 2-day meeting with lead STACK developer Chris Sangwin was held at Loughborough University in December 2013, where the basic architecture and principles of the question type were laid out. In the following months, the ILIAS question type was created².

The developers set up a Moodle installation with STACK, examined the code and analysed how it works. Then the 'library-like' Moodle core was extracted and integrated into the template of an ILIAS question plugin. The techniques used followed the open source nature of STACK and were supported by the fact that STACK uses the same technology (PHP/MySQL) as ILIAS.

Initially, the only way to add questions was to create them in Moodle, then import the moodleXML file into ILIAS.

Results

From the beginning, the ILIAS community has shown a great interest in using the integration of STACK. To date, nine separate institutions have participated in the crowd funding of the plugin. Over time, several workshops have been organized by the community. A German user manual and workshops to teach the creation of STACK questions was created at FAU and ported to an online module by the University of Göttingen. In 2018, the first international STACK conference³ was held at the Innovation in Learning Institute, bringing together around 65 participants from the Moodle and ILIAS communities.

As hoped, STACK is now being used extensively at FAU for innovative e-assessment. The ILIAS installation at FAU contains around 3000 questions from over 100 different authors. Additionally, a local 'STACK user group' was founded with 40 members from various disciplines, including mathematics, natural sciences, engineering, and economics.

Based on external and internal funding, it is possible to permanently provide new major versions of the plugin for each major ILIAS release every year, as well as several minor bug fix versions in between. The core of STACK used by the plugin is also updated every year. The ILIAS questions are fully compatible with the Moodle version and can be exchanged between the two Learning Management Systems through the moodleXML format.

² The question type can be accessed here: <https://github.com/ilifau/assStackQuestion>

³ More information about the first international STACK conference can be found on <https://www.stack-konferenz.de>

Aufgabe Cosh23 (Terme)

Vereinfachen Sie den Ausdruck
 $A = 5 \cdot a \cdot b - (b \cdot (a - 6) + 8 \cdot b).$

$A =$ ✓

$5 \cdot a \cdot b - a \cdot b + 6 \cdot b - 8 \cdot b$ interpreted as:
 $5 \cdot a \cdot b - a \cdot b + 6 \cdot b - 8 \cdot b$

Hinweis: Der gesuchte Ausdruck enthält keine Klammern.

Check

Ihre Antwort ist nicht korrekt.
 Sie können den Ausdruck noch weiter vereinfachen. Der Term darf keine Klammern enthalten.

Figure: A typical ILIAS question about simplification.

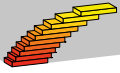
Challenges

The biggest challenge for the initial integration of STACK into ILIAS was the decoupling of all dependencies from Moodle in the STACK core. Currently, around 50 patches are maintained for that. Furthermore, there were major differences in ILIAS and Moodle regarding the definition and display of feedback and various test player options. The completely different GUIs of Moodle and ILIAS meant the whole authoring interface of STACK had to be redesigned.

When the community of STACK users in ILIAS grew, the number of bug reports related to exotic edge cases of STACK usage also grew. This could, for example, be issues relating to questions with a huge number of inputs. Furthermore, an increasing community of users brought in very different opinions about the styles of a question, its validation and feedback. Dealing with these issues was a significant challenge.

A barrier that had to be evaluated is time. The initial work in integrating STACK into ILIAS took significant effort, and the project has to be continually maintained as bug fixes and project features are applied. It is estimated that, on average, 70% of a full-time employee's time is required to maintain the STACK developments for ILIAS. This is currently partially funded by the ILIAS community and the Innovation in Learning Institute.

Server performance was an issue when it came to using STACK in large institutions. Maxima was designed as a desktop application, and so lacks a set of good practice recommendations for hardware and configuration options depending on the expected load. To solve this, there are some initiatives such as the GoMaxima project from HAW Hamburg and an initiative at the University of Göttingen for a docker swarm of Maxima pools.



Enablers

The meeting with the lead STACK developer was a major help in getting the project off on the right track. The comprehensive STACK documentation was also invaluable for understanding the question type.

Additionally, the support from the ILIAS special interest group was invaluable. This group helped sketch the authoring interface in ILIAS, write documentation and organise the first workshops. It also helped moderate the discussion of feature requests from the ILIAS community and organise crowd funding for the maintenance of the plugin. Furthermore, since the Innovation in Learning Institution is not a mathematics institution, it was invaluable to get help from mathematicians to write good demonstration questions.

Finally, the port was supported by the common technology of ILIAS and STACK and the fact Moodle and ILIAS have very similar quiz structures, and both treat STACK as a question plugin type. Implementation differences aside, the main principles are the same.

What's Next?

The ILIAS question type will continue to be updated as STACK, Moodle and ILIAS get updated. In particular, the way question types in ILIAS are implemented will change in a future ILIAS version, which means the question type will have to be adapted accordingly. There are also plans to make STACK less dependent on the Moodle library. If this is successful, the ILIAS plugin will have to be largely rewritten (since it depends on the Moodle library), however the developers are optimistic. From writing the question type the first time around, they have gained invaluable experience. Furthermore, the authoring interface will not have to be rewritten.

There are many wishes from the ILIAS community for new features to be implemented. Some of these are step-wise feedback, as well as more control of the feedback style, for example the colour of feedback boxes. These may be added to the STACK question type in the future.

Finally, the maintenance relies heavily on crowdfunding managed between institutions (typically the Innovation in Learning Institution and another University). This means contracts have to be written between the heads of each institution, which can be difficult to manage. There are therefore considerations to transfer the maintenance of the question type to an ILIAS service provider, as it would then be easier to get crowdfunding contracts from Universities.

STACK at Scale: The Open University

The Open University



Tim Lowe, School of Mathematics and Statistics, Tim Hunt, Information Technology.

Introduction

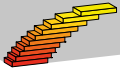
The Open University is the UK's largest academic institution. In 2017/18 there were approximately 175,000 students studying with the University, mainly part time, making approximately 65,000 full-time equivalent students. Students combine their study with work, family, caring and other responsibilities. Students study from home, at a distance, guided by the University's Moodle-based VLE and using a combination of online and printed materials. Students are supported by a tutor who provides individual support to a group of typically 20 students and usually offers a combination of face-to-face and online tutorial support.

The School of Mathematics and Statistics offers a number of undergraduate qualifications in mathematics (with a total intake of approximately 1300 each year) and a taught MSc, which is the largest such course in the UK. In addition, students throughout the University can take one or more mathematics modules in support of their main subject of study. There are approximately 13,000 student-module combinations within mathematics and statistics each year.

STACK is an ideal tool to support the distance-learning of mathematics, as it enables students to practice key techniques from home whilst receiving immediate feedback on their answers. Students can attempt different randomised variants of questions to support their learning and development.



Figure: The Open University.



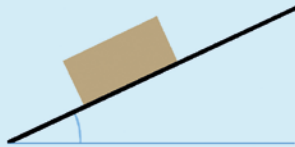
Execution

STACK was first used in the curriculum in 2014 with the launch of a new introductory calculus module: MST124 “Essential Mathematics I”, which is currently studied by approximately 2,800 students per year. Since then, STACK has been taken up by many modules at all levels. The main use is for formative “practice quizzes” allowing students to practice the important techniques taught. At lower levels, it is also used for summative assignments, which both encourage students to use the formative quizzes to prepare for the assignment and practice their mathematics, and help students keep on pace with their study by providing deadlines. At the postgraduate level, a series of linked-STACK questions has been developed to guide students through more complex mathematical arguments in the calculus of variations [1].

The School of Mathematics and Statistics have been supporting colleagues in the School of Engineering and Innovation who use STACK to support their teaching of mathematically-based topics. The School is also using STACK to support students between formal module study, and in preparation for study as a key component of online “Revise and Refresh” support materials.

STACK is currently used in at least 10 modules providing a total of 330 CATS credits and reaching over 6,500 students annually. Over one million STACK questions are answered by students each year, which is approximately 18% of all quiz questions answered by OU students.

A box of mass 25 kg is at rest on a rough plane inclined at an angle of 20° to the horizontal.



Calculate the magnitude of the friction force, in newtons, to two significant figures.

(Take the magnitude of the acceleration due to gravity, g , to be 9.8 m s^{-2} .)

The magnitude of the friction force is N (to 2.s.f.).

Check

Figure: A typical OU STACK question.

Server

STACK is supported by the University Information Technology department as part of the main Moodle installation. This comprises of 11 load-balanced Apache Web Servers and a large PostgreSQL database. The Maxima computations are handed on two separate MaximaPool servers [2]. Despite the high level of use described above, STACK has always performed robustly and we have suffered no unplanned service outages. All authors are strongly encouraged to create question tests for each STACK question. This enables the IT department to upgrade to new versions of STACK with confidence. The tests for all questions are run with the new version of STACK and any problems this reveals are addressed before the upgrade is applied to the live system.

Feedback

Students' engagement with the quizzes is often demonstrated by comments posted in online forums, for example questioning details of the worked solution provided. Students are often not content with just being able to answer every question in a quiz, but continue to try to correctly answer the quiz in the minimum time.

Student feedback on the use of STACK quizzes has been positive, with many choosing to praise them in the unprompted open comments of the university module feedback survey. Conversely, negative comments are received where a module does not have STACK quizzes. Typical student comments include the following:

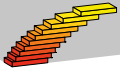
- ❖ "I do value the practice quizzes because you do get practice, the feedback explanations are very good and detailed."
- ❖ "[I] have done two runs of the practice quiz which has made a big difference for me and will become something I will be doing much more as [it] helps so much."
- ❖ "I have been doing the practice quiz every morning now for 2 weeks ... I managed 100% on one of the quizzes and over 90% now on all of them so my confidence level is going up!"
- ❖ "The practice quizzes were excellent and I think absolutely essential for consolidating your understanding of the subject."

What's Next

The School currently plans to introduce STACK quizzes into additional modules as they are updated. Future plans include extending the use of STACK to further, higher level modules and to expand the use of STACK to support students in checking their preparedness for the study of various modules.

References

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STACK for Engineering Mathematics and the Abacus Material Bank

Antti Rasila, Guangdong Technion
– Israel Institute of Technology



Abstract

Since 2006, Aalto University has been using STACK to run online assessments within their Engineering Mathematics courses. Initially, the University was running a STACK version developed in-house specifically for the University, but this was later merged back into the main STACK branch in version 3.0. After receiving funding, the material bank Abacus was created, with the purpose of helping institutions collaborate and share STACK questions. The project's agreement was written in a way that ensured question authors kept their copyright, while still encouraging collaboration. Abacus has 30 members and its courses cover most of undergraduate mathematics and many parts of physics.

Motivation

The MatTa group (“Matematiikkaa tietokoneavusteisesti”; “mathematics by using computers”) was founded in 1990s by lecturer Emeritus Simo K. Kivelä at the Department of Mathematics and Systems Analysis at Aalto University, formerly known as Helsinki University of Technology. The purpose of the group was to investigate the use of computers and information technology to teach engineering mathematics. Initially, the projects involved, for example, using visualisations, multimedia and symbolic computation to make mathematical content more interesting and accessible for students. The group also produced a number of free Finnish language electronic lecture notes with interactive content and a substantial database of traditional pen and paper exercise assignments called Euler. In 2006, the group started to

take interest in automatic assessment, as they believed computer-aided methodologies could achieve a practical impact in teaching activities there. There was also hope at the University that Computer Aided Assessment could reduce the number of students assistants required to grade homework, both to save cost and because a sufficient number of qualified graders was not always available. Initially, the group focused on the Maple T.A. system, however it was found lacking in several aspects. The most important of these were performance issues and an incompatibility with various browsers, in particular ones available for the Linux operating system. The high licensing fees were also a problem, as they could undo all the savings that the University hoped to achieve by using the system.

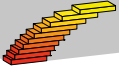
In spring of 2006, the group started gaining interest in STACK, which has an open source license. This license would allow the University to use its in-house software development skills to improve the platform where needed, and it would also guarantee that licence fees would not be imposed in the future. After deciding to use STACK, which was running its standalone version 1.x at the time, the group spent the summer of 2006 modifying the software to better suit the University's needs. This included replacing the rendering code of mathematical formulas to support browsers other than Microsoft Internet Explorer, translating the software to Finnish and a Shibboleth¹ based integration with the Finnish national Haka single-sign-in system.

Implementing STACK

After modifying the system, the University started testing STACK. The initial test was with the university's Basic Course in Engineering Mathematics KP3. A. Rasila wrote four STACK assignments for each week of the course, all of which were relatively simple questions involving numerical or algebraic input. The system had numerous small technical issues, but the students were generally happy with the system as evidenced both by student feedback and general usage statistics [1].

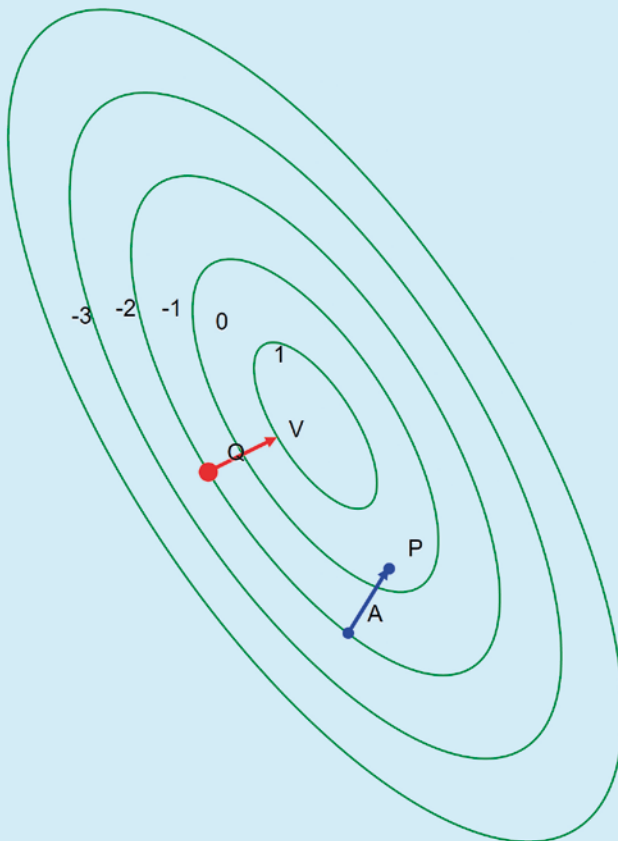
Encouraged by the positive results, it was decided to test the system more comprehensively to convince the teaching community that automatic assessment was worth of the effort in teaching engineering mathematics. It was important to show that the online assessment could be used by teachers with little programming skill, and that it would not cause a large increase in their workload. This was crucial for the long-term success of the project, as MatTa's previously developed materials were often only used by a handful of teachers, most of whom where developers themselves. Furthermore, the group needed to demonstrate that the system had real cost savings, as well as good learning outcomes compared to traditional types of assignments. The University set up an experimental course "Discrete Mathematics", where STACK would form a significant portion of the final grade [2]. Following the positive results of this course, the use of STACK began to spread at the University, and within three years, most basic courses in Engineering Mathematics at Aalto University were using STACK.

¹ See <https://www.shibboleth.net/>



In the figure below five contour lines of the function $f(x, y)$ are drawn. Move the point P such that the vector from A to P and the gradient of f at A are parallel with each other. (The problem is about the direction, not the length of the vectors.)

Next, move also the point Q such that the gradient of f at Q and the vector $\mathbf{V} = [1.777 \quad 0.917]$ are parallel with each other.



Check

Figure: A STACK question where the configuration of the diagram is the answer.

STACK Developments

A key ingredient in the success of STACK at Aalto University were the in-house developments by programmer Matti Harjula, who also wrote his Master's Thesis on this work [3]. His initial work included analysing the data of the first 2006 experiment and further developing the system to address any shortcomings. His project was successful, as it led to a system that many lecturers at the department were willing to use, and no major issues were identified in the subsequent use of the system.

However, it also led to a new problem. Independently from Aalto University, the British STACK developers had been developing STACK 2.0, which was very different from the 1.x series Aalto University had been using for their work. The in-house developments by Aalto University were not easily implemented into STACK 2.0, but merging the Aalto University code back into the trunk of STACK was still necessary for a number of reasons. Firstly, a unified code base would speed up the overall development of STACK, and the Department at Aalto University did not intend to permanently commit many resources to maintaining the system. Secondly, there was interest from other Finnish universities to try the system, however they preferred to use the "official" British version of STACK, which would prevent them from using the materials developed at Aalto University. Because of this, the Finnish and British developers agreed to join forces in combining their work into STACK 3.0. During the STACK 3.0 merge, experimental features of STACK were also being added to the Aalto University version, see [4]. These features were related to several pedagogical ideas, [5,6,7].

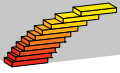
Creating Abacus



There were two key advancements in 2015. Firstly, STACK 3.0 was completed and Aalto University had finalised their plans to deploy the system as a replacement to their old in-house version. Since the University was now using the official STACK version, it was easier to collaborate with other Universities on STACK material. Secondly, the consortium of the seven Finnish technology universities called for proposals in developing collaborative practices in education. A draft proposal for joint development of electronic materials for mathematics education was proposed and selected as a pilot project with over half a million euros of funding. A. Rasila was chosen as the national coordinator of this project, which became known as the Abacus² consortium [8].

Abacus is a database of teaching materials, mainly STACK questions, covering most basic topics in undergraduate mathematics. Its mission statement is to help lecturers find high quality and free learning materials, achieved through facilitating the sharing of online teaching resources between institutions. National and international collaboration is at the heart of the project, as it helps standardise the platform, minimise the necessary contribution of each institution and contribute to the overall market share of the platform. Language differences between courses is not a concern, since adding translations to individual questions are almost trivial compared to the expense of actually programming the mathematics e-content.

² ABACUS is available at <https://abacus.aalto.fi/>



In drafting the initial consortium agreement, Abacus drew from two other related projects, namely a 2009 e-learning collaboration project with the Bavarian Virtual University VHB, which was funded by the later discontinued Finnish Virtual University, and the MatTa material bank Euler.

Fair Use

In projects largely developed through public and institutional funding, fair use is a common problem. Some partners may feel they are contributing more than the rest, and hence will not be interested in collaboration and instead push for in-house development. Furthermore, collaborators who develop materials without being paid are often unhappy when someone else makes even minor monetary gains from their work, which has led to a widespread use of non-commercial licenses for academic work. While understandable, these restrictions hinder the acceptance of platforms like Abacus, and lead to divisions in the community. Furthermore, non-commercial licenses can be confusing, as the exact nature of commercial activity is not clearly defined in academia. For example, is “free” work done by a salaried lecturer commercial use? Finally, it can be necessary for institutions to share the sources of assignments due to international differences in notation, course content and language. On the other hand, a pure open source model is not particularly attractive for e-assessment materials, since students should not be able to search online for the teachers’ answers.

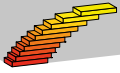
To solve these problems, the Abacus consortium agreement was written such that there are no strict legal restrictions on what kind of institutions can join the consortium or what kinds of materials can be included. The materials are shared in a way such that each partner gets substantial rights to develop them based on a license similar to open source, but is not allowed to share the source code with third parties. The original developers retain their copyright of the materials and are allowed to give them other licenses besides the one required by Abacus. Since new members are reviewed by the consortium’s steering group and are required to make a one-time contribution to the consortium, the agreement encourages collaboration and guarantees that existing partners benefit from the growth of the consortium.

Outlook

As of July 2019, the Abacus consortium consists of 30 members, 17 of which are from Finland, as well as three from Estonia and two from both Germany and Norway. Other countries present in the consortium are China, France, Ireland, Portugal, Ukraine, and the United Kingdom. There are ongoing discussions with several other potential partners, mainly from Europe. The database currently contains problem assignments for most bachelor level courses in mathematics, as well as a substantial number of physics assignments. Although Abacus accepts contributions in any language, most materials are available in English. Translations of the remaining Finnish-only materials are ongoing, and are expected to be completed in 2020.

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Promoting STACK Across Disciplines at Loughborough University

Loughborough University

Ian Jones



**Loughborough
University**

Mathematics
Education Centre

Abstract

At Loughborough University, STACK has been implemented across many disciplines. Courses are created by module leaders, who prepare paper-based problems, and academics and students who implement these problems in STACK. To promote STACK to lecturers across the University, a showcase module page was developed that makes it easy to navigate and browse good STACK examples across disciplines.

Introduction

STACK was installed at Loughborough University in 2014. Since then, a programme of development and dissemination led by Loughborough's Mathematics Education Centre has resulted in widespread use across the University. The University has encouraged multidisciplinary take up beyond the mathematics department, and now 57 modules across six degree programmes include STACK content. STACK is used for assessment in topics including mathematics, business, statistics, chemical engineering, physics and foundation programmes.

Execution

Initially, questions were developed in conjunction with module leaders. Typically a module leader provided the developers with paper-based problem sheets. The developers, who included academics, as well as PhD and undergraduate mathematics students working on projects, adapted the problems into STACK questions. The example below illustrates how the developers paid special attention to making use of STACK's unique feedback feature to provide personalised feedback to students.

Exercises

1. For the following functions find $\frac{\partial f}{\partial x}$ and $\frac{\partial f}{\partial y}$
- (a) $f(x, y) = x + 2y + 3$
 - (b) $f(x, y) = x^2 + y^2$
 - (c) $f(x, y) = x^3 + xy + y^3$
 - (d) $f(x, y) = x^4 + xy^3 + 2x^3y^2$
 - (e) $f(x, y, z) = xy + yz$

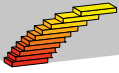
For the following function find $\frac{\partial f}{\partial x}$ and $\frac{\partial f}{\partial y}$.

$$f(x, y) = y^2 + x^2.$$

$\frac{\partial f}{\partial x} =$

$\frac{\partial f}{\partial y} =$

Figure: A paper-based question and its STACK equivalent.



Alongside development, the STACK content was promoted and disseminated to lecturers across the University via internal workshops. This included workshops to train module leaders in developing their own content. Additionally, a module page was developed, viewable to all teaching staff, that acts as a showcase for STACK content.

STACK: online mathematics assessment

General

Module Noticeboard

Welcome to STACK 	Exporting and Importing Questions 	Syntax quiz
Algebra 	Arithmetic 	Business
Complex numbers 	Differentiation 	Engineering

Figure: The University developed a showcase module page.

The showcase module page is designed to be easy to use and navigate, even for beginners. This is because newcomers often find navigating, selecting and exporting questions off-putting. To overcome this, all questions within a topic have been wrapped into a downloadable .zip file, and only a few navigable example questions have been provided. The download screen for business-based STACK questions is shown below. Business questions have been a particular success, and a new project to develop further STACK content for the School of Business and Economics begins in Autumn 2019.

Complex numbers

This is an example of a quiz made up of STACK complex number test items. Includes complex arithmetic, polar coordinates, Aargand diagrams, finding cubic roots.

Complex numbers quiz

Contains four example complex numbers items, some of which are multipart.

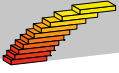
Complex numbers (31 questions) 266.6KB application/xml

Download this file and import into your module page to have access to the complex number questions.

Figure: The download screen for a collection of business STACK questions.

What's Next?

STACK development continues at Loughborough. The University continues to be particularly interested in promoting STACK in areas where it is not commonly used. This development includes developing STACK content for degree programmes beyond mathematics, such as statistics for psychologists, as well as areas within mathematics that are traditionally difficult to assess automatically. A particularly exciting area being explored is assessing proof comprehension, and an example prototype question is shown below.



This is a proof that $\sqrt{2}$ is irrational.

1. Assume that $\sqrt{2}$ is rational.
2. Let $\sqrt{2} = \frac{m}{n}$ where $m > 0$ and $n > 0$ are coprime integers,
3. so $2 = \frac{m^2}{n^2}$
4. $\Leftrightarrow 2n^2 = m^2$
5. $\therefore m^2$ is divisible by 2.
6. It follows that m is divisible by 2.
7. Let $m = 2t$ where t is a positive integer,
8. we have $4t^2 = 2n^2 \Leftrightarrow n^2 = 2t$,
9. so n^2 is divisible by 2
10. and so is n .
11. But m and n being divisible by 2 is a contradiction that they are both coprime.
12. Therefore $\sqrt{2}$ is irrational.

a) The proof mentions *coprime* numbers. Give an examples of a number that is *coprime* to 60.

b) Line 10 concludes the proof by claiming that the fact m and n are even is a direct contradiction to an earlier statement, which line is that statement on?

c) This proof can be used to show that other square roots ($\sqrt{\alpha}$) are also irrational. Give an example of another value of alpha that this proof could prove to be irrational without anyway changing the structure of the proof in any way.

Check

Figure: A STACK question assessing some skills in proof comprehension.

Institutional Support for STACK in Edinburgh

University of Edinburgh

Interview with Chris Sangwin



THE UNIVERSITY
of EDINBURGH

Abstract

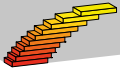
At the University of Edinburgh, online assessment has been consolidated with in-house support, mostly with STACK, for the majority of year one and two mathematics modules and for many other mathematics and general science courses. A dedicated learning-technologist post was created to help course organisers implement online assessment. Replacing the current online assessment with human marking is currently estimated to save the School over 6100 hours of work marking students' work each year.

Motivation

The School of Mathematics at the University of Edinburgh had, over the years, grown to use a number of separate online assessment systems. However, there was growing concern that using separate systems created a number of problems, for example students had to learn the syntax of many different systems. It was also problematic that students needed to purchase access to the online systems provided by publishers. At the same time, student numbers in the UK continued to grow [1], and there were growing expectations from students for more frequent formative assessment in their courses. This motivated the school to bring all assessments in-house with STACK.

Execution

STACK was first used in Edinburgh at a small scale for the Lothian Equal Access Programme for Schools (LEAPS) summer school in 2016. Following the success with LEAPS, STACK was used in larger courses such as Introduction to Linear Algebra, a year one module with over 600 students, in the 2016-17 academic year. The University's primary learning environment is Blackboard "LEARN", so students gain access to a dedicated STACK service via the LTI protocol through their normal LEARN pages.



Having identified the need for additional support, the School of Mathematics created a dedicated “Learning Technologist” post. The primary goal of this post was to transform existing (largely paper-based) problem sets into online assessments and to develop alternatives to existing external online assessments. The Technologist used the following process to transform these assessments.

- ❖ Before the year started, current forms of assessment were reviewed, and suitable STACK questions were written together with course organisers.
- ❖ During the academic year, quizzes were made available, and their usage monitored.
- ❖ When the year was over, this monitoring data was analysed and questions were updated where appropriate.

Creating online assessments is significant additional work, but once the quizzes have been created they require minimal work to maintain and can last for the lifetime of the course. Because of this cost-benefit relationship, employing a Learning Technologist has been an effective strategy in ensuring change actually took place.

Give an example of a matrix with eigenvalues $\{-7, 6\}$.

1	2
2	1

Your last answer was interpreted as follows:

$$\begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix}$$

Check

✘ Incorrect answer.
We have calculated the eigenvalues of your matrix as $\{-1, 3\} \neq \{-7, 6\}$.
Marks for this submission: 0.00/1.00. This submission attracted a penalty of 0.10.

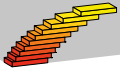
Figure: A typical Introduction to Linear Algebra question.

Results

In 2019, many courses have online assessments using STACK. A course will typically have weekly or fortnightly assessment, many using a combination of flipped classroom “reading quizzes” (RQ), formative “practice quizzes” (PQ) and summative “assessed quizzes” (AQ). Some courses also make an optional mock online exam available to students. The following table provides an overview from the 18/19 year, including the number of students, how often quizzes were given out (per week W or semester S), the average number of questions per quiz (#Q/quiz) and the total number of questions (#Qs).

Year	Course	Students	Quizzes	#Q/quiz	#Qs
1	Introduction to Linear Algebra	604	2 RQ/W	2-3	
			1 AQ/W	6-8	110
1	Calculus and its Applications	594	1 RQ/W	4	
			1 AQ/W	10	150
1	Proofs and Problem Solving	344	1 RQ/W	2	20
1	Engineering Mathematics 1a	393	} 3 PQ/W	5	
	Mathematics for Natural Sciences 1a	138		} 1 AQ/W	7-10
1	Engineering Mathematics 1b	401	} 3 PQ/W	3-5	
	Mathematics for Natural Sciences 1b	136		} 1 AQ/W	7-10
1	Mathematics for Physics 1	193	5 AQ/S	5	25
1	Mathematics for Physics 2	202	5 AQ/S	6	30
1	Fundamentals of Algebra and Calculus	113	5 PQ/W	10-20	
			3 AQ/W	6-12	950
2	Probability	312	1 AQ/W	3-5	36
2	Several Variable Calculus and DEs	287	1 PQ/W	5-12	
			1 AQ/W	5-10	180
3	Honours Algebra	202	9 AQ/S	1-8	46
3	Combinatorics and Graph Theory	60	2 RQ/W	2	
			4 AQ/S	1-6	45
3	Symmetry and Geometry	36	1 AQ/W	4-7	45
4	Galois Theory	27	1 RQ/W	2	22
PG	Fundamentals of Optimization	190	3 AQ/S	4-8	16
PG	Introduction to Probability and Statistics	23	1 PQ/W	6-8	
			5 AQ/S	4	62

Table: Courses using STACK assessments at The University of Edinburgh 2018-19.



Cost

Over the course of this project, the Learning Technologist estimates that developing a fully functioning quiz of 8 questions took about two person-days, or 16 hours of work, to create. As an example, consider the large first-year course “Introduction to Linear Algebra”, with around 600 students in 65 tutorial groups. Replacing half of the weekly hand-ins with online assessments has saved each tutor over one hour of marking per week. Hence, over 65 hours of work is saved each week as a result. In addition, the students now complete more than double the number of practice problems, providing them with enhanced formative feedback which would be impossible to resource otherwise. Since online quizzes can be reused each year, this will be a consistent saving for as long as the course remains.

Overall, it is estimated that STACK saves the University over 6100 hours of work each year [2].



Figure: Many School of Mathematics courses are taken at the James Clerk Maxwell Building.

Barriers

The goal was to have the first 4-6 weeks of quizzes ready before the beginning of a semester. However, as the term progressed, it was not unusual for the last few quizzes to be ready only “at the last moment”. This did not give the question authors a lot of time to review their questions, but does reflect the realities of teaching.

It was also important that the course organiser were fully involved in the authoring process. They had to give clear and explicit guidance on the learning objectives and help review the mathematical content.

Finally, when working with large question banks, organising became difficult. Large Moodle question banks are tricky to browse, and maintenance becomes tedious when questions are duplicated between similar courses.

Enablers

The most significant factor in the success of this project was the dedicated Learning Technologist post. This was essential, as it assured course organisers had practical support, and that online quizzes could have a consistently high level of quality.

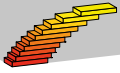
It was helpful to be able to work closely with STACK developers when designing quizzes, in a way that is difficult with non-open-source programs. For example, the “numerical” input type which helps students enter answers at a pre-specified level of numerical accuracy, was designed to help cope with the difficulties of assessing vague student answers in statistics questions [3].

What’s Next?

The School of Mathematics continues to expand the use STACK at the University of Edinburgh. Future challenges involve modifying STACK to suit more conceptual courses, such as group theory and real analysis, as well as expanding into statistics and computer programming using the CodeRunner system [4]. There are also plans for wider use of STACK in summative assessments, with the possibility of using it in online examinations.

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